ST7103 THEORY OF ELASTICITY AND PLASTICITY

UNIT I

ELASTICITY

PART-A

1.Define principal plane

2.Define principal stress

3.Give the equations of equilibrium

4.Define Hook's law

5.What are Lame's constants?

6.What are compatibility equations?

7.Give the biharmonic equation for the case of plane strain

8.Give the equations denoting stress strain relations

9.Give the navier equations

10.Give the equations relating stress-strain

11.What are the boundary conditions in terms of stresses?

12.Define plane stress

13. Express the biharmonic equation for plane stress

14.What are stress invariants?

15.What are strain invariants?

16.What are Beltrami-michell equations?

17.Express the boundary conditions in terms of displacements along the horizontal direction

18.What are stress formulations?

19.What are displacement formulations?

20.What are stress tensors?

PART-B

1.The stress field in a body is given by

$\sum x=0.005z$; $\gamma xy=0.003xy$

$\sum y=0.001x$; $\gamma yz=-0.001xz$

$\sum z=-0.002xy$; $\gamma zx=0.001y$

Check whether it is a compatible strain field

2.The displacement field in a homogeneous isotropic elastic body is given by

$u=k\{(3x^2z+60x)i+(5z^2+10xy);+(6z^2+2xyz)k\}$,

where $k=1x10^{-6}$mm,if $E=2x10^5 N/mm^2$, r=0.25,Evaluate the stress components at a point P(5,10,-15)mm.

3.Prove that the biharmonic equation for the plane stress condition is

Delta4=$d^4/dx^4 + 2(d^4/dx^2dy^2) + d^4/dy^4$.

4.The state of stress at a point wih respect to the xyz system is

3	2	-2	
2	0	-1	
-2	-1	2	kN/m^2

Determine the stress tensor relative to the x'y'z' coordinate system obtained by a rotation through 30° about the z-axis.

5.The state of stress at a particular point relative to the xyz coordinate system is given by the stress matrix

15	10	-10	
10	10	0	
-10	0	40	M pa

Determine the normal stress and the magnitude and direction of the shear stress on a surface intersecting the point is parallel to the plane given by the equations

2x-y+3z=9

6.For the stress tensor given below,determine the principal stresses and the direction cosines associated with the normals to the surface of each principal stress.

3000	1000	1000
1000	0	2000
1000	2000	0 N/m²

7.The stress components at a point are given by the following array:

10	5	6
5	8	10
6	10	6 Mpa

Calculate the principal stresses and principal planes.

8.The state of stress at a point is given by $\sigma x=100, \sigma y=200, \sigma z=-100, \tau xy=-200, \tau yz=100, \tau xz=-300$kpa.

Determine

a)The stress invariants

b)The principal stresses

c)The direction cosines of the principal planes

9.The displacement field in a body is specified as:

$Ux=(x^2+3)x10^{-3}$

$Uy=3y^2z \ x \ 10^{-3}$

$Uz=(x+3z)x10^{-3}$

Determine the strain components at appoint whose coordinates are (1,2,3)

10.The state of strain at a point is given by

$\sum x=0.001$, $\sum y=-0.003$, $\sum z=0.002$,

$\gamma xy=0.001$, $\gamma yz=0.0005$, $\gamma xz=0.002$

Determine the strain invariants and the principal strains.

UNIT-II

ELASTICITY SOLUTON

PART-A

1.Express the stress function in the form of a doubly infinite power series.

2.Express the stress compatibility equation for plane stress case.

3.Express the stress compatibility equation for plane strain case.

4.Give the strain displacement relations applying hooke's law.

5.Give the equations relating to bending of a beam by uniform load.

6.Give the Cauchy-riemann equations

7.What are conjugate harmonic functions?

8.Give the property of the analytic functions.

9.give the orthogonal ltrajectories of the curve.
10.Give the equation of complex displacement.

11.Express the displacement and stress configurations in terms of complex potentials.

12.How does the stress function satisfies the biharmonic equation?

13.State Cauchy integral theorem.

14.Give the relationship between Cartesian and polar co-ordinates.

15.What are the assumptions made in Winkler's Theroy?

16.Express the stresses acting in a Tapering beam.

17.Express the stresses acting in an infinite wedge.

18.Express the equilibrium equations in polar co-ordinates.

19.Express the stress components in terms of an Airy stress function

20.What is a Mellin transform?

PART-B

1.Derive the deflection equation for the bending of a cantilever loaded at the end in terms of Cartesian coordinates.

2. Derive the deflection equation for bending a simply supported beam uniformly loaded over the entire span interms of Cartesian coordinates

3.Prove that the following Airy's stress functions and examine the stress distribution represented by them:

a)$\phi=Ax^2+By^2$, b)$\phi=Ax^3$,c)$^3\phi=A(x^4-3x^2y^2)$

4.Show that the Airy's stress function $\phi=A(xy^3-3/4 \ xyh^2)$ represents stress distribution in a cantilever beam loaded at the free end with load P.find the value of A if $\tau xy=0$ at $y=+- h/2$ where b and h are width and depth respectively of the cantilever.

5.Derive the two-dimentional biharmoic equation in terms of cartersian coordinates.

6. Derive the two-dimentional biharmoic equation in terms of polar coordinates.

7.Prove that $\sigma=P/A +M/AR \ [1+y/m(R+y)]$ according to Winkler's Theory.

8.A steel gun barrel is subjected to an internal pressure of 70MPa.The internal diameter of the barrel is 75mm and external diameter of 225mm.A steel band 25mm thick and internal diameter 0.075mm smaller than the external diameter of the gun barrel is shrunk on the gun barrel. Calculate

a)The shrinkage pressure on the gun barrel,

b)The maximum stress in the steel band.

c)Minimum temperature to which the band must be heated to make the assembly.

For steel E=200GPa, v=0.3 and coefficient of thermal expansion=$10x10^{-6}$/°C

9.A cast iron shaft 500mm diameter is rotating at 3000rev/min.Its density is 747 kg/m³ and poinson's ratio 0.28.It is constrained at its ends so that it cannot expand or contract axially.calculate the total longitudinal thrust over its entire cross section due to centrifugal stresses.

10.A steel turbine rotor of 750mm outer diameter ,150mm inner diameter and 50mm thickness has 100 blades 150mm long, each weighing 4N.It is shrink-fitted on a rigid shaft.Calculate the initial shrinkage allowance on he inner diameter of the rotor so that it just loosens on the shaft at 3000rev/min.Take E=200GPa,v=0.3.The density of shaft and rotor is 7500kg/m³ .

UNIT-3

TORSION OF NON-CIRCULAR SECTION

PART-A

1.State st.Venant's Theory.

2.Give the poison's equation of prismatic bars of non-circular cross- sections.

3.Give the Bredt's formula

4.Give the Cauchy-Reimann equation in terms of conjugate harmonic function

5.Give the complex torsion function.

6.Give the Prandtl equation

7.Give the Green's formula

8.What are the analogous method?

9.Give the poison's equation relating to torsion

10.What are the lines of shearing stress.

11.What is a Warping function?

12.Give the equation of resultant shear stress at a point.

13.Give the torsion equation

14.Gives the equation of stesses relating to torsion of elliptical cross- section bar.

15.Give the displacement equation relating to torsion of elliptical cross- section bar.

16.Give the equation of angle of trust relating to torsion of elliptical cross- section bar.

17.Give the equaton of torsional rigidity relating to torsion of elliptical cross- section bar.

18.What is a membrance analogy?

19.Give the expression for angle of trust for a thin-wall hollow section.

20.Give the equation of the trusting couple of a conical shaft.

PART-B

1.Derive the torque equation of a prismatic bar subjected to turst T,according to st.Venant's theory.

2.A square shaft rotating at 250rpm,transmits torque to a crane which is designed to lift maximumload of 150 kN at a speed of 10m/min.If the efficiency of crane gearing is 65% estimate the size of the shaft for the maximum permissible shear stress of 35MPa.Also calculate the angle of turst of the shaft for a length of 3m.Take G=100 GPa.

3.A 300mm steel beam with flanges and web 12.5mm thick ,flange widh 300mm is subjected to a torque of 4 kN m.Find the maximum shear and angle of turst per unit length.G=100 GPa.

4.Derive the torsion equation of a hollow cylinder.

5.Show that in the torsion effect of thin-walled tubes, the ratio of rate of twists approaches junity.

6.Derive the torsion equation for hollow bars.

7.Derive the torque equation of a thini rectangular section.

8.Derive the torsion equation of thin-walled hollow section.

9.A hollow circular torsion member has an outside diameter of 22mmm nad inside diameter of 18mm, with mean diameter ,D=20mm AND t/D=0.10.calculate the torque and angle of twist per unit length if shearing stress at mean diameter is 70MPa. Calculate these values if a out is made through the wall thickness along the entire length G=77.5 GPa.

10.Apply the conformal mapping to an epitrochoidal section and derive the torsional rigidity.

UNIT-4

ENERGY METHODS

PART-A

1.State maxwell-Betti Reciprocal Theorem

2.StateCastigliano's Theorem

3.State the principle of virtual work

4.State the principle of potential energy

5. State the principle of complementary energy

6.What is a dummy load method?

7.What is Rayleigh's method?

8.Give the assumption in Rayleigh-Ritz Method.

9.What is finite difference method?

10.Give the finite difference equations.

11.Give the advantage of finite element method.

12.What is finite element concept?

13.What are the factors to be considered while selecting displacement models?

14.Give the equilibrium relation between the element stiffnessmatrix and nodal force vector.

15.Give the general form of a polynomial.

16.Give the elastic relationship between stresses and strain.

17.Give the conditions for convergence criteria.

18.What are isoparametric elements?

19.Give the Jacobian matrix in terms of Cartesian curvilinear derivatives.

20.What is a shape function?

PART-B

1.Derive the expression for torsion of a rectangular bar by Rayleigh-Ritz Method.

2.Derive the expression for deflection of a simply supported beam with udl over the entire span by Rayleigh-Ritz Method.

3.Derive the expression for deflection of a rectangular plate by the principle of virtual work.

4.Derive the expression for strain energy of a rectangular plate by Rayleigh-Ritz method.

5.Analyze the torsion of a bar of square section usingfinite difference technique.

6.Explain the finite element concept in detail.

7.Calculate the reaction at A for the spring supported cantilever subjected to udl over the entire span by strain energy method.

8.Derive the expression for deflection of a cantilever beam by Rayleigh's method.

9.Derive the expression for the rotation at A of a simply supported beam AB with udl over the entire span.

10.Calculate the strain energy for a simple supported rectangular beam AB of span 10m with 1kN point load at the centre of the span .The beam size is 40x60mm.

UNIT-5

PLASTICITY

PART-A

1.State Saint-Venant’s theory.

2.State Newton’s law.

3,State PrandtbRews theory.

4.Give Kelvin’s relation.

5.GiveMaxwell’s relation.

6.What is meant by yield criteria.

7.Give the yield conditions.

8.Give the assumptions in plastic analysis.

9.Explain soap film analogy.

10.Explain sand Heap Analogy.

11.What are residual stresses in plastic bending?

12.Define shape factor.

13.What is a plastic hinge?

14.Define load factor.

15.What are residual stresses in torsion.

16.State St.Venant’s theory for torsion.

17.Define twisting moment.

18.Give the Tresca’s yield criteria.

19.Give the Von-Mises yield criteria.

20.Explain the auto frottage phenomenon.

PART-B

1.Discuss in detail the various theories of failure normally adopted to find the yield criteria.

2.A rectangular beam having linear stress-strain behavior is 6cm wide and 8cm deep.It is 3m long,simply supported at the ends and carries a uniformly distributed load over the whole span.The load is increased so that the outer 2cm depth of the beam yields plastically.If the yield stress for the beam material is 240MPa,plot the residual stress distribution in the beam.

3.A rectangular-section beam has adepth of 20cm and a width of 10cm.The beam is made of a steel with identical properties in tension and compression.The material has a yield stress σ_0=315MPa,E=210GPa,H=700MPa.The beam has yielded upto a depth of 5cm. Determine the magnitude of bending moment applied to the beam.

4.A circular shaft of inner radius 4cm and outer radius 10cm is subjected to a twisting couple so that the outer 2cm deep shell yields plastically.Determine the twisting couple applied to the shaft .yield stress in shear for the shaft material is 425N/mm^2.Also determine the couple for full yielding.

5.Detail the experimental verification of st.Venant's Theory of plastic flow in detail.

6.A solid circular shaft of radius 12cm is subjected to transmit 600 k W at 540rpm.The maximum torque is 30 percent greater than the mean torque.If the shear stress strain curve for the shaft materials is given by $\tau=280\gamma$^0.25,determine the maximum stress induced in the shaft and the corresponding angle of twist.What would be these values if the shear stress-strain curve is a linear one?G=0.84x10^5 N/mm^2.

7.A hollow circular shaft of inner radius 2cm and outer radius 5cm is subjected to a twisting moment so that the outer 1cm deep shall yields plastically.The yield stress in shear for the shaft material is 175 MPa and it is made of a non-linear material whose shear stress-shear strain curve is given by $\tau=280\gamma$^0.25.If this twisting moment is now released,determine the residual stress distribution in the shaft and the associated residual angle of twist .G=0.84x10^5 N/mm^2.

8.A thick cylinder of internal radius 15cm and external radius 25cm is subjected to an internal pressure P MPa.If the yield stress for the cylinder material is 220N/mm^2,determine

a)the pressure at which the cylinder will start yielding just at the inner radius b)the stresses when the cylinder has a plastic front radius of 20cm and c)the stresses when whole of the cylinder has yielded. Assume Von-Mises yield condition is a state of plane strain.

9.A solid circular shaft of 8cm radius is subjected to a twisting couple so that the outer 3cm deep shell of the shaft yieldsplastically.If the yield stress in shear for the shaft material is 150MPa,determine the value of twisting couple applied and the associated angle of twist. G=0.84x10^5 N/mm^2.

10.A hollow circular shaft of inner radius 5cm and outer radius 10cm is s ubjected to a twisting couple of 5000Nm .If the shear stress-strain diagram for the shaft material is given by $\tau=350\ \gamma$^0.3,determine the maximum shear stress induced in the shaft and the angle of twist per unit length.