

**CE 1302 – STRUCTURAL ANALYSIS –
CLASSICAL METHODS
B.E. III Yr Civil Engineering**

Prepared by

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UNIT-I DEFLECTION OF DETERMINATE STRUCTURES

1. Why is it necessary to compute deflections in structures?

Computation of deflection of structures is necessary for the following reasons:

- a. If the deflection of a structure is more than the permissible, the structure will not look aesthetic and will cause psychological upsetting of the occupants.
- b. Excessive deflection may cause cracking in the materials attached to the structure. For example, if the deflection of a floor beam is excessive, the floor finishes and partition walls supported on the beam may get cracked and unserviceable.

2. What is meant by cambering technique in structures?

Cambering is a technique applied on site, in which a slight upward curve is made in the structure/ beam during construction, so that it will straighten out and attain the straight shape during loading. This will considerably reduce the downward deflection that may occur at later stages.

3. Name any four methods used for computation of deflections in structures.

- | | |
|------------------------------|--|
| 1. Double integration method | 2. Macaulay's method |
| 3. Conjugate beam method | 4. Moment area method |
| 5. Method of elastic weights | 6. Virtual work method- Dummy unit load method |
| 7. Strain energy method | 8. Williot Mohr diagram method |

4. State the difference between strain energy method and unit load method in the determination of deflection of structures.

In strain energy method, an imaginary load P is applied at the point where the deflection is desired to be determined. P is equated to zero in the final step and the deflection is obtained.

In unit load method, an unit load (instead of P) is applied at the point where the deflection is desired.

5. What are the assumptions made in the unit load method?

1. The external & internal forces are in equilibrium.
2. Supports are rigid and no movement is possible.
3. The materials is strained well with in the elastic limit.

6. Give the equation that is used for the determination of deflection at a given point in beams and frames.

Deflection at a point is given by,

$$\delta_I = \int_0^l \frac{M_x m_x dx}{EI}$$

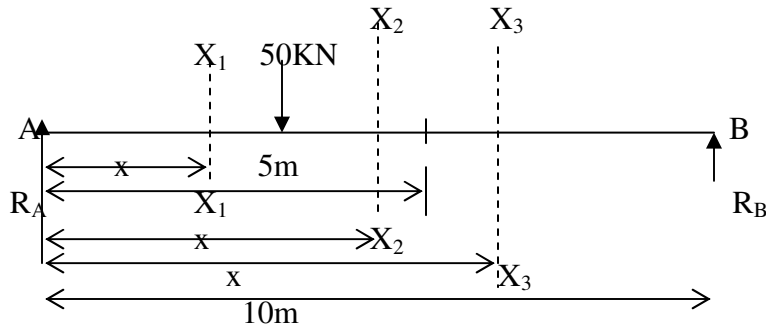
where M_x = moment at a section X due to the applied loads

m_x = moment at a section X due to a unit load applied at that point I and in the direction of the

desired displacement

EI = flexural rigidity

7. Write down the equations for moments due to the external load for beam shown in Fig.



Portion	M_x	Limits
AC	$R_A x$	0 to 4
CD	$R_A x - 50(x-4)$	4 to 5
DB	$R_A x - 50(x-4)$	5 to 10

8. Distinguish between pin jointed and rigidly jointed structure.

Sl.no	Pin jointed structure	Rigidly jointed structure
1.	The joints permit change of angle between connected member.	The members connected at a rigid joint will maintain the angle between them even under deformation due to loads.
2.	The joints are incapable of transferring any moment to the connected members and vice-versa.	Members can transmit both forces and moments between themselves through the joint.
3.	The pins transmit forces between connected member by developing shear.	Provision of rigid joints normally increases the redundancy of the structures.

9. What is meant by thermal stresses?

Thermal stresses are stresses developed in a structure/member due to change in temperature. Normally, determine structures do not develop thermal stresses. They can absorb changes in lengths and consequent displacements without developing stresses.

10. What is meant by lack of fit in a truss?

One or more members in a pin jointed statically indeterminate frame may be a little shorter or longer than what is required. Such members will have to be forced in place during the assembling. These are called members having Lack of fit. Internal forces can develop in a redundant frame (without external loads) due to lack of fit.

11. Write down the two methods of determining displacements in pin jointed plane frames by the unit load concept.

The methods of using unit loads to compute displacements are,

- i) dummy unit load method.
- ii) using the principle of virtual work.

12. What is the effect of temperature on the members of a statically determinate plane truss.

In determinate structures temperature changes do not create any internal stresses. The changes in lengths of members may result in displacement of joints. But these would not result in internal stresses or changes in external reactions.

13. Distinguish between 'deck type' and 'through type' trusses.

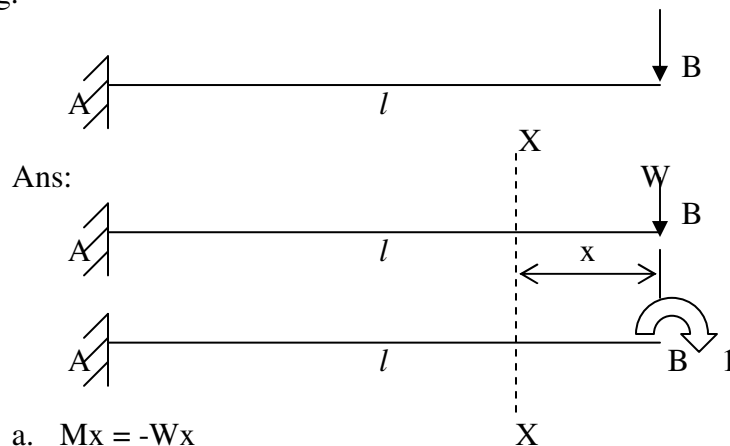
A deck type truss is one in which the road is at the top chord level of the trusses. We would not see the trusses when we ride on the road way.

A through type truss is one in which the road is at the bottom chord level of the trusses. When we travel on the road way, we would see the web members of the trusses on our left and right. That gives us the impression that we are going 'through' the bridge.

14. Define static indeterminacy of a structure.

If the conditions of statics i.e., $\Sigma H=0$, $\Sigma V=0$ and $\Sigma M=0$ alone are not sufficient to find either external reactions or internal forces in a structure, the structure is called a statically indeterminate structure.

15. Briefly outline the steps for determining the rotation at the free end of the cantilever loaded as shown in Fig.



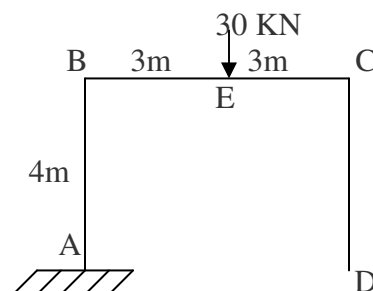
a. $M_x = -Wx$

b. $m_x = -1$

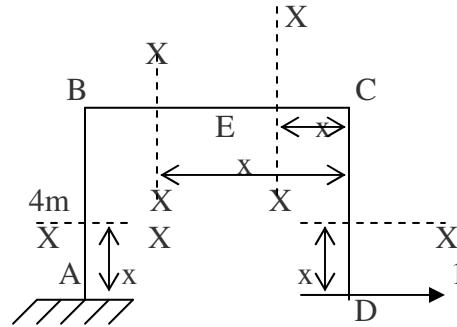
c.

d. $\theta_B = \int_0^l \frac{M_x m_x dx}{EI} = \int_0^l \frac{(-Wx)(-1) dx}{EI}$

16. The horizontal displacement of the end D of the portal frame is required. Determine the relevant equations due to the unit load at appropriate point.



Ans:



Apply unit force in the horizontal direction at D. m_x values are tabulated as below:

Portion	m_x	Limits
DC	$1x$	0 to 4m
CE	1×4	0 to 3m
EB	1×4	3 to 6m
BA	$1x$	0 to 4m

17. Differentiate the statically determinate structures and statically indeterminate structures?

Sl.No	statically determinate structures	statically indeterminate structures
1.	Conditions of equilibrium are sufficient to analyze the structure	Conditions of equilibrium are insufficient to analyze the structure
2.	Bending moment and shear force is independent of material and cross sectional area.	Bending moment and shear force is dependent of material and independent of cross sectional area.
3.	No stresses are caused due to temperature change and lack of fit.	Stresses are caused due to temperature change and lack of fit.

18. Define : Trussed Beam.

A beam strengthened by providing ties and struts is known as Trussed Beams.

19. Define: Unit load method.

The external load is removed and the unit load is applied at the point, where the deflection or rotation is to found.

20. Give the procedure for unit load method.

1. Find the forces P_1, P_2, \dots in all the members due to external loads.
2. Remove the external loads and apply the unit vertical point load at the joint if the vertical deflection is required and find the stress.
3. Apply the equation for vertical and horizontal deflection.

UNIT-II INFLUENCE LINES

1. Where do you get rolling loads in practice?

Shifting of load positions is common enough in buildings. But they are more pronounced in bridges and in gantry girders over which vehicles keep rolling.

2. Name the type of rolling loads for which the absolute maximum bending moment occurs at the midspan of a beam.

(i) Single concentrated load (ii) udl longer than the span (iii) udl shorter than the span (iv) Also when the resultant of several concentrated loads crossing a span, coincides with a concentrated load then also the maximum bending moment occurs at the centre of the span.

3. What is meant by absolute maximum bending moment in a beam?

When a given load system moves from one end to the other end of a girder, depending upon the position of the load, there will be a maximum bending moment for every section. The maximum of these bending moments will usually occur near or at the midspan. The maximum of maximum bending moments is called the absolute maximum bending moment.

4. Where do you have the absolute maximum bending moment in a simply supported beam when a series of wheel loads cross it?

When a series of wheel loads crosses a simply supported beam, the absolute maximum bending moment will occur near midspan under the load W_{cr} , nearest to midspan (or the heaviest load). If W_{cr} is placed to one side of midspan C, the resultant of the load system R shall be on the other side of C; and W_{cr} and R shall be equidistant from C. Now the absolute maximum bending moment will occur under W_{cr} . If W_{cr} and R coincide, the absolute maximum bending moment will occur at midspan.

5. What is the absolute maximum bending moment due to a moving udl longer than the span of a simply supported beam?

When a simply supported beam is subjected to a moving udl longer than the span, the absolute maximum bending moment occurs when the whole span is loaded.

$$M_{\max \max} = \frac{wl^2}{8}$$

6. State the location of maximum shear force in a simple beam with any kind of loading.

In a simple beam with any kind of load, the maximum positive shear force occurs at the left hand support and maximum negative shear force occurs at right hand support.

7. What is meant by maximum shear force diagram?

Due to a given system of rolling loads the maximum shear force for every section of the girder can be worked out by placing the loads in appropriate positions. When these are plotted for all the sections of the girder, the diagram that we obtain is the maximum shear force diagram. This diagram yields the 'design shear' for each cross section.

8. What is meant by influence lines?

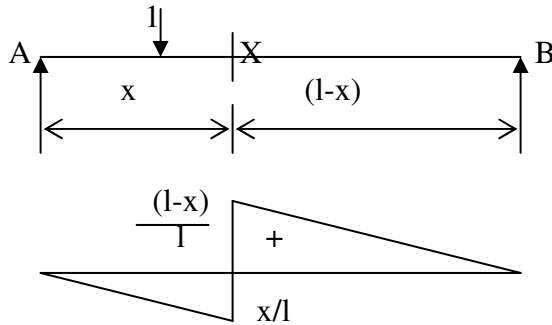
An influence line is a graph showing, for any given frame or truss, the variation of any force or displacement quantity (such as shear force, bending moment, tension, deflection) for all positions of a moving unit load as it crosses the structure from one end to the other.

9. What are the uses of influence line diagrams?

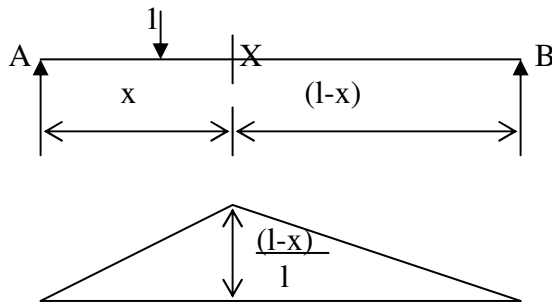
(i) Influence lines are very useful in the quick determination of reactions, shear force, bending moment or similar functions at a given section under any given system of moving loads and

- (ii) Influence lines are useful in determining the load position to cause maximum value of a given function in a structure on which load positions can vary.

10. Draw the influence line diagram for shear force at a point X in a simply supported beam AB of span 'l' m.



11. Draw the ILD for bending moment at any section X of a simply supported beam and mark the ordinates.



12. What do you understand by the term reversal of stresses?

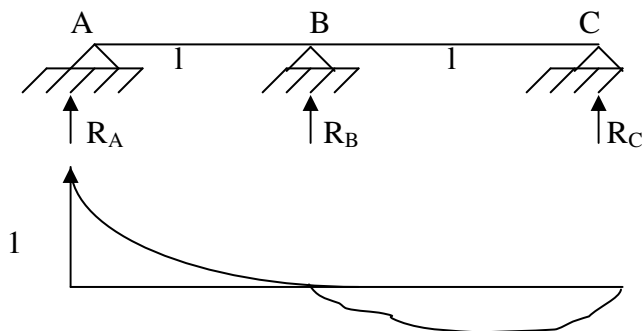
In certain long trusses the web members can develop either tension or compression depending upon the position of live loads. This tendency to change the nature of stresses is called reversal of stresses.

13. State Muller-Breslau principle.

Muller-Breslau principle states that, if we want to sketch the influence line for any force quantity (like thrust, shear, reaction, support moment or bending moment) in a structure,

- (i) We remove from the structure the resistant to that force quantity and
- (ii) We apply on the remaining structure a unit displacement corresponding to that force quantity. The resulting displacements in the structure are the influence line ordinates sought.

14. State Maxwell-Betti's theorem.



In a linearly elastic structure in static equilibrium acted upon by either of two systems of external forces, the virtual work done by the first system of forces in undergoing the displacements caused by the second system of forces is equal to the virtual work done by the second system of forces in undergoing the displacements caused by the first system of forces.

Maxwell Betti's theorem helps us to draw influence lines for structures.

15. What is the necessity of model analysis?

- (i) When the mathematical analysis of problem is virtually impossible.
- (ii) Mathematical analysis though possible is so complicated and time consuming that the model analysis offers a short cut.
- (iii) The importance of the problem is such that verification of mathematical analysis by an actual test is essential.

16. Define similitude.

Similitude means similarity between two objects namely the model and the prototype with regard to their physical characteristics:

- Geometric similitude is similarity of form
- Kinematic similitude is similarity of motion
- Dynamic and/or mechanical similitude is similarity of masses and/or forces.

17. State the principle on which indirect model analysis is based.

The indirect model analysis is based on the Muller Breslau principle.

Muller Breslau principle has led to a simple method of using models of structures to get the influence lines for force quantities like bending moments, support moments, reactions, internal shears, thrusts, etc.

To get the influence line for any force quantity, (i) remove the resistant due to the force, (ii) apply a unit displacement in the direction of the (iii) plot the resulting displacement diagram. This diagram is the influence line for the force.

18. What is the principle of dimensional similarity?

Dimensional similarity means geometric similarity of form. This means that all homologous dimensions of prototype and model must be in some constant ratio.

19. What is Begg's deformeter?

Begg's deformeter is a device to carry out indirect model analysis on structures. It has the facility to apply displacement corresponding to moment, shear or thrust at any desired point in the model. In addition, it provides facility to measure accurately the consequent displacements all over the model.

20. Name any four model making materials.

Perspex, plexiglass, acrylic, plywood, sheet araldite and bakelite are some of the model making materials. Micro-concrete, mortar and plaster of paris can also be used for models.

21. What is 'dummy length' in models tested with Begg's deformeter.

Dummy length is the additional length (of about 10 to 12mm) left at the extremities of the model to enable any desired connection to be made with the gauges.

22. What are the three types of connections possible with the model used with Begg's deformeter.

- (i) Hinged connection
- (ii) Fixed connection
- (iii) Floating connection

23. What is the use of a micrometer microscope in model analysis with Begg's deformeter.

Micrometer microscope is an instrument used to measure the displacements of any point in the x and y directions of a model during tests with Begg's deformeter.

UNIT-III ARCHES

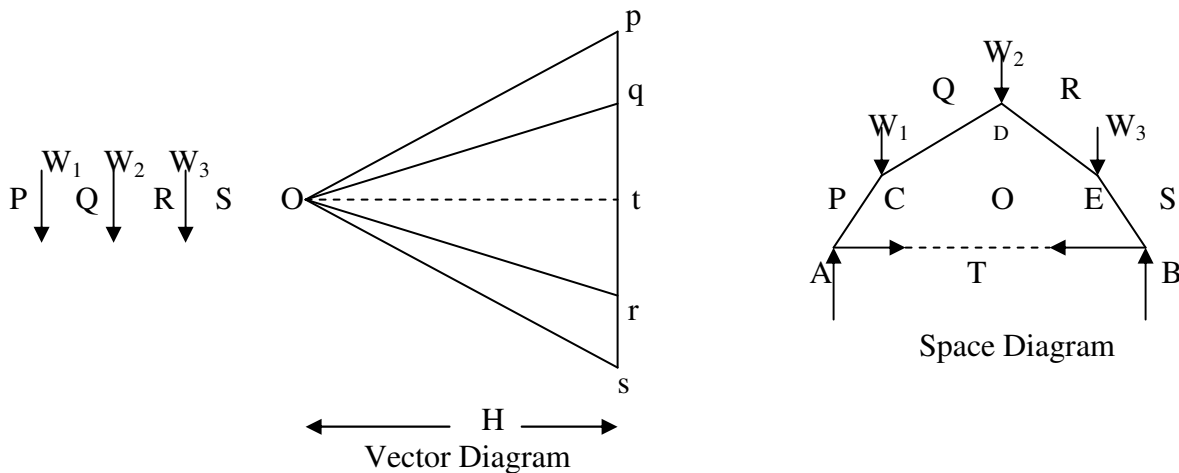
1. What is an arch? Explain.

An arch is defined as a curved girder, having convexity upwards and supported at its ends.

The supports must effectively arrest displacements in the vertical and horizontal directions. Only then there will be arch action.

2. What is a linear arch?

If an arch is to take loads, say W_1 , W_2 , and W_3 (fig) and a Vector diagram and funicular polygon are plotted as shown, the funicular polygon is known as the linear arch or theoretical arch.



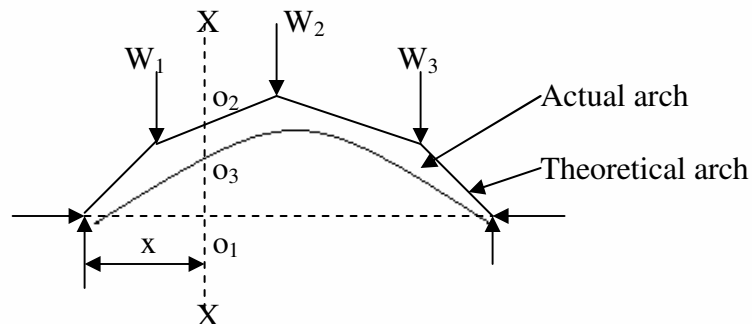
The polar distance 'ot' represents the horizontal thrust. The links AC, CD, DE, and EB will be under compression and there will be no bending moment. If an arch of this shape ACDEB is provided, there will be no bending moment.

For a given set of vertical loads W_1 , W_2etc., we can have any number of linear arches depending on where we choose 'O' or how much horizontal thrust (ot) we choose to introduce.

3. State Eddy's theorem.

Eddy's theorem states that "The bending moment at any section of an arch is proportional to the vertical intercept between the linear arch (or theoretical arch) and the centre line of the actual arch."

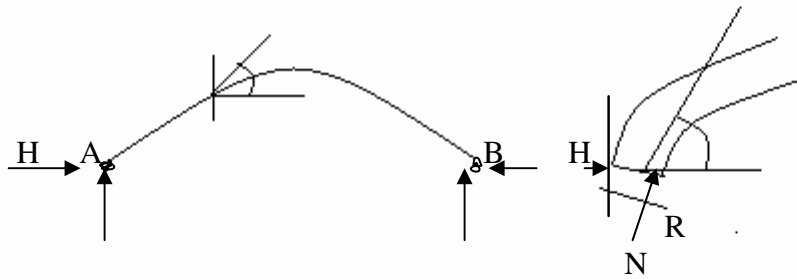
$$BM_x = \text{Ordinate } O_2O_3 \times \text{scale factor}$$



4. What is the degree of static indeterminacy of a three hinged parabolic arch?

For a three hinged parabolic arch, the degree of static indeterminacy is zero. It is statically determinate.

5. Explain with the aid of a sketch, the normal thrust and radial shear in an arch rib.



Let us take a section X of an arch. (fig (a)). Let θ be the inclination of the tangent at X. If H is the horizontal thrust and V the vertical shear at X, from the free body of the RHS of the arch, it is clear that V and H will have normal and radial components given by,

$$N = H \cos\theta + V \sin\theta$$

$$R = V \cos\theta - H \sin\theta$$

6. Which of the two arches, viz. circular and parabolic is preferable to carry a uniformly distributed load? Why?

Parabolic arches are preferable to carry distributed loads. Because, both, the shape of the arch and the shape of the bending moment diagram are parabolic. Hence the intercept between the theoretical arch and actual arch is zero everywhere. Hence, the bending moment at every section of the arch will be zero. The arch will be under pure compression which will be economical.

7. What is the difference between the basic action of an arch and a suspension cable?

An arch is essentially a compression member which can also take bending moments and shears. Bending moments and shears will be absent if the arch is parabolic and the loading uniformly distributed.

A cable can take only tension. A suspension bridge will therefore have a cable and a stiffening girder. The girder will take the bending moment and shears in the bridge and the cable, only tension.

Because of the thrusts in the cables and arches, the bending moments are considerably reduced.

If the load on the girder is uniform, the bridge will have only cable tension and no bending moment on the girder.

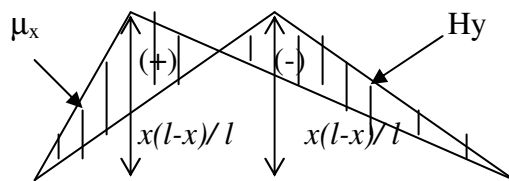
8. Under what conditions will the bending moment in an arch be zero throughout.

The bending moment in an arch throughout the span will be zero, if

(i) the arch is parabolic and (ii) the arch carries uniformly distributed load throughout the span.

9. Draw the ILD for bending moment at a section X at a distance x from the left end of a three hinged parabolic arch of span 'l' and rise 'h'.

$$M_x = \mu_x - Hy$$



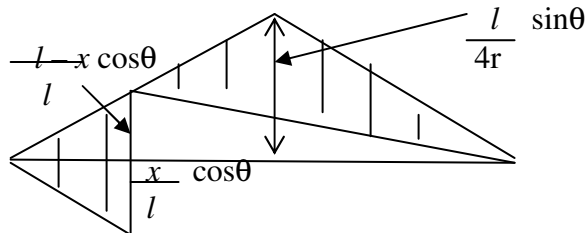
10. Indicate the positions of a moving point load for maximum negative and positive bending moments in a three hinged arch.

Considering a three hinged parabolic arch of span 'l' and subjected to a moving point load W, the position of the point load for

- a. Maximum negative bending moment is $0.25l$ from end supports.
- b. Maximum positive bending moment is $0.211l$ from end supports.

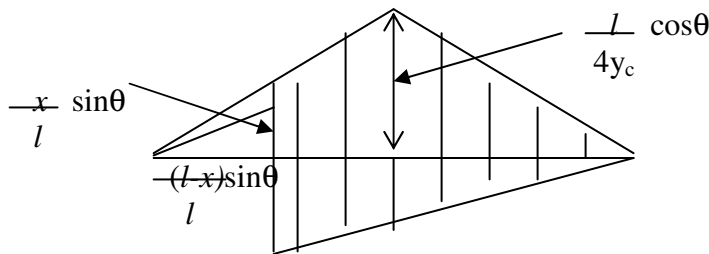
11. Draw the influence line for radial shear at a section of a three hinged arch.

Radial shear is given by $F_x = H \sin\theta - V \cos\theta$,
where θ is the inclination of tangent at X.



12. Sketch the ILD for the normal thrust at a section X of a symmetric three hinged parabolic arch.

Normal thrust at X is given by $P = H \cos\theta + V \sin\theta$,
where θ is the inclination of tangent at X.



13. Distinguish between two hinged and three hinged arches.

Sl.No.	Two hinged arches	Three hinged arches
1.	Statically indeterminate to first degree	Statically determinate
2.	Might develop temperature stresses	Increase in temperature causes increase in central rise. No stresses.
3.	Structurally more efficient	Easy to analyse. But in construction, the central hinge may involve additional expenditure.
4.	Will develop stresses due to sinking of supports	Since this is determinate, no stresses due to support sinking.

14. Explain rib-shortening in the case of arches.

In a two hinged arch, the normal thrust which is a compressive force along the axis of the arch will shorten the rib of the arch. This in turn will release part of the horizontal thrust. Normally, this effect is not considered in the analysis (in the case of two hinged arches).

Depending upon the importance of the work we can either take into account or omit the effect of rib shortening. This will be done by considering (or omitting) strain energy due to axial compression along with the strain energy due to bending in evaluating H.

15. Explain the effect of yielding of support in the case of an arch.

Yielding of supports has no effect in the case of a 3 hinged arch which is determinate. These displacements must be taken into account when we analyse 2 hinged or fixed arches under

$$\frac{\partial U}{\partial H} = \Delta H \quad \text{instead of zero}$$

$$\frac{\partial U}{\partial V_A} = \Delta V_A \quad \text{instead of zero}$$

Here U is the strain energy of the arch and ΔH and ΔV_A are the displacements due to yielding of supports.

16. Write the formula to calculate the change in rise in three hinged arch if there is a rise in temperature.

$$\text{Change in rise} = \left(\frac{l^2 + 4r^2}{4r} \right) \alpha T$$

where l = span length of the arch
 r = central rise of the arch
 α = coefficient of thermal expansion
 T = change in temperature

17. In a parabolic arch with two hinges how will you calculate the slope of the arch at any point.

$$\text{Slope of parabolic arch} = \theta = \tan^{-1} \left(\frac{4r}{l^2} (l - 2x) \right)$$

where θ = Slope at any point x (or) inclination of tangent at x .
 l = span length of the arch
 r = central rise of the arch

18. How will you calculate the horizontal thrust in a two hinged parabolic arch if there is a rise in temperature.

$$\text{Horizontal thrust} = \frac{l \alpha T E I}{\int_0^l y^2 dx}$$

where l = span length of the arch
 y = rise of the arch at any point x
 α = coefficient of thermal expansion
 T = change in temperature
 E = Young's Modulus of the material of the arch
 I = Moment of inertia

19. What are the types of arches according to the support conditions.

- i. Three hinged arch
- ii. Two hinged arch
- iii. Single hinged arch
- iv. Fixed arch (or) hingeless arch

20. What are the types of arches according to their shapes.

- i. Curved arch
- ii. Parabolic arch
- iii. Elliptical arch
- iv. Polygonal arch

UNIT-1V SLOPE-DEFLECTION METHOD

1. What are the assumptions made in slope-deflection method?

- (i) Between each pair of the supports the beam section is constant.
- (ii) The joint in structure may rotate or deflect as a whole, but the angles between the members meeting at that joint remain the same.

2. How many slope deflection equations are available for a two span continuous beam?

There will be 4 nos. of slope-deflection equations, two for each span.

3. What is the moment at a hinged end of a simple beam?

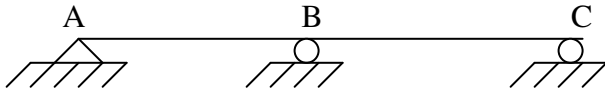
Moment at the hinged ends of a simple beam is zero.

4. What are the quantities in terms of which the unknown moments are expressed in slope-deflection method?

In slope-deflection method, unknown moments are expressed in terms of

- (i) slopes (θ) and (ii) deflections (Δ)

5. The beam shown in Fig. is to be analysed by slope-deflection method. What are the unknowns and, to determine them, what are the conditions used?



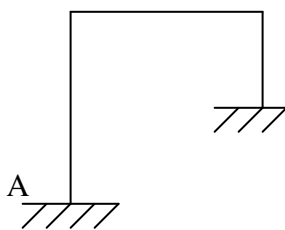
Unknowns: $\theta_A, \theta_B, \theta_C$

Equilibrium equations used: (i) $M_{AB} = 0$ (ii) $M_{BA} + M_{BC} = 0$ (iii) $M_{CB} = 0$

6. How do you account for sway in slope deflection method for portal frames?

Because of sway, there will be rotations in the vertical members of a frame. This causes moments in the vertical members. To account for this, besides the equilibrium, one more equation namely shear equation connecting the joint-moments is used.

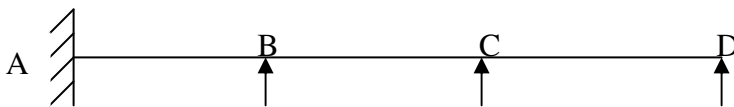
7. Write down the equation for sway correction for the portal frame shown in Fig.



The shear equation (sway correction) is

$$\frac{M_{AB} + M_{BA}}{l} + \frac{M_{CD} + M_{DC}}{l} = 0$$

8. Write down the slope deflection equation for a fixed end support.

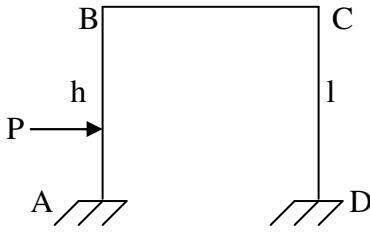


The slope deflection equation for end A is $M_{AB} = M'_{AB} + \frac{2EI}{l} \left[2\theta_A + \theta_B + \frac{3\Delta}{l} \right]$

Here $\theta_A = 0$. Since there is no support settlement, $\Delta = 0$.

$$M_{AB} = M'_{AB} + 2EI \left[\theta_B + \frac{3\Delta}{l} \right]$$

9. Write down the equilibrium equations for the frame shown in Fig.

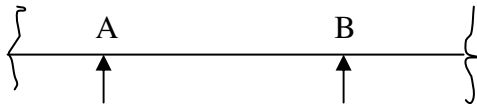


Unknowns : θ_B, θ_C
 Equilibrium equations : At B, $M_{BA} + M_{BC} = 0$
 At C, $M_{CB} + M_{CD} = 0$
 Shear equation : $\frac{M_{AB} + M_{BA} - Ph}{l} + \frac{M_{CD} + M_{DC}}{l} + P = 0$

10. Who introduced slope-deflection method of analysis?

Slope-deflection method was introduced by Prof. George A. Maney in 1915.

11. Write down the general slope-deflection equations and state what each term represents?



General slope-deflection equations:

$$M_{AB} = M'_{AB} + \frac{2EI}{l} \left[2\theta_A + \theta_B + \frac{3\Delta}{l} \right]$$

$$M_{BA} = M'_{BA} + \frac{2EI}{l} \left[2\theta_B + \theta_A + \frac{3\Delta}{l} \right]$$

where, M'_{AB}, M'_{BA} = Fixed end moment at A and B respectively due to the given loading.
 θ_A, θ_B = Slopes at A and B respectively
 Δ = Sinking of support A with respect to B

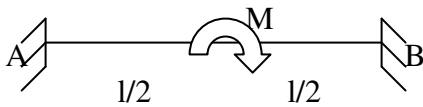
12. Mention any three reasons due to which sway may occur in portal frames.

Sway in portal frames may occur due to (i) unsymmetry in geometry of the frame (ii) unsymmetry in loading or (iii) Settlement of one end of a frame.

13. How many slope-deflection equations are available for each span?

Two numbers of slope-deflection equations are available for each span, describing the moment at each end of the span.

14. Write the fixed end moments for a beam carrying a central clockwise moment.



Fixed end moments : $M'_{AB} = M'_{BA} = \frac{M}{4}$

15. State the limitations of slope deflection method.

- (i) It is not easy to account for varying member sections
- (ii) It becomes very cumbersome when the unknown displacements are large in number.

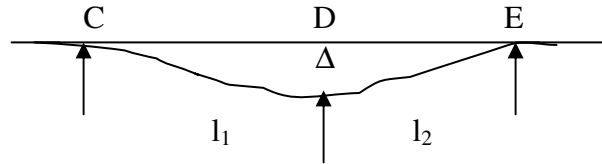
16. Why is slope-deflection method called a 'displacement method'?

In slope-deflection method, displacements (like slopes and displacements) are treated as unknowns and hence the method is a 'displacement method'.

17. Define degrees of freedom.

In a structure, the number of independent joint displacements that the structure can undergo are known as degrees of freedom.

18. In a continuous beam, one of the support sinks. What will happen to the span and support moments associated with the sinking of support.



Let support D sink by Δ . This will not affect span moments. Fixed end moments (support moments) will get developed as under

$$M'_{CD} = M'_{DC} = -6EI \frac{\Delta}{l_1^2}$$

$$M'_{DE} = M'_{ED} = -6EI \frac{\Delta}{l_2^2}$$

19. A rigid frame is having totally 10 joints including support joints. Out of slope-deflection and moment distribution methods, which method would you prefer for analysis? Why?

Moment distribution method is preferable.

If we use slope-deflection method, there would be 10 (or more) unknown displacements and an equal number of equilibrium equations. In addition, there would be 2 unknown support moments per span and the same number of slope-deflection equations. Solving them is difficult.

20. What is the basis on which the sway equation is formed for a structure?

Sway is dealt with in slope-deflection method by considering the horizontal equilibrium of the whole frame taking into account the shears at the base level of columns and external horizontal forces.

The shear condition is
$$\frac{M_{AB} + M_{BA}}{l} - Ph + \frac{M_{CD} + M_{DC}}{l} + P = 0$$

UNIT-V MOMENT DISTRIBUTION METHOD

1. What is the difference between absolute and relative stiffness?

Absolute stiffness is represented in terms of E , I and l , such as $4EI/l$.

Relative stiffness is represented in terms of I and l , omitting the constant E . Relative stiffness is the ratio of stiffness to two or more members at a joint.

2. Define: Continuous beam.

A Continuous beam is one, which is supported on more than two supports. For usual loading on the beam hogging (- ive) moments causing convexity upwards at the supports and sagging (+ ve) moments causing concavity upwards occur at mid span.

3. What are the advantages of Continuous beam over simply supported beam?

1. The maximum bending moment in case of continuous beam is much less than in case of simply supported beam of same span carrying same loads.

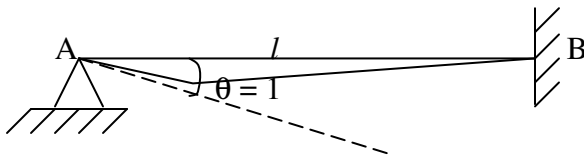
2. In case of continuous beam, the average bending moment is lesser and hence lighter materials of construction can be used to resist the bending moment.

4. In a member AB, if a moment of -10 KNm is applied at A, what is the moment carried over to B?

Carry over moment = Half of the applied moment

\therefore Carry over moment to B = $-10/2 = -5$ KNm

5. What are the moments induced in a beam member, when one end is given a unit rotation, the other end being fixed. What is the moment at the near end called?



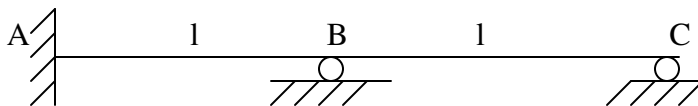
When $\theta = 1$,

$$M_{AB} = \frac{4EI}{l}$$

$$M_{BA} = \frac{2EI}{l}$$

M_{AB} is the stiffness of AB at B.

6. A beam is fixed at A and simply supported at B and C. $AB = BC = l$. Flexural rigidities of AB and BC are $2EI$ and EI respectively. Find the distribution factors at joint B if no moment is to be transferred to support C



Joint B: Relative stiffness: $\frac{I_+}{l} = \frac{2I}{l}$ for BA. $K_{BA} = 2$

$\frac{3}{4} \times \frac{I_+}{l} = \frac{3I}{4l}$ for BC $K_{BC} = \frac{3}{4} = 0.75$

Distribution factors:

$$\text{DF for BA: } \frac{K_{BA}}{K_{BA} + K_{BC}} = \frac{2}{2 + 0.75} = \frac{8}{11} = 0.727$$

$$\text{DF for BC: } \frac{K_{BC}}{K_{BC} + K_{BA}} = \frac{0.75}{2 + 0.75} = \frac{3}{11} = 0.273$$

7. Define: Moment distribution method.(Hardy Cross method).

It is widely used for the analysis of indeterminate structures. In this method, all the members of the structure are first assumed to be fixed in position and fixed end moments due to external loads are obtained.

8. Define: Stiffness factor.

It is the moment required to rotate the end while acting on it through a unit rotation, without translation of the far end being

(i) Simply supported is given by $k = 3 EI / L$

(ii) Fixed is given by $k = 4 EI / L$

where, E = Young's modulus of the beam material.

I = Moment of inertia of the beam

L = Beam's span length.

9. Define: Distribution factor.

When several members meet at a joint and a moment is applied at the joint to produce rotation without translation of the members, the moment is distributed among all the members meeting at that joint proportionate to their stiffness.

Distribution factor = Relative stiffness / Sum of relative stiffness at the joint

If there is 3 members, Distribution factors = $\frac{k_1}{k_1 + k_2 + k_3}$, $\frac{k_2}{k_1 + k_2 + k_3}$, $\frac{k_3}{k_1 + k_2 + k_3}$

10. Define: Carry over moment and Carry over factor.

Carry over moment: It is defined as the moment induced at the fixed end of the beam by the action of a moment applied at the other end, which is hinged. Carry over moment is the same nature of the applied moment.

Carry over factor (C.O) : A moment applied at the hinged end B “ carries over” to the fixed end A, a moment equal to half the amount of applied moment and of the same rotational sense. C.O =0.5

11. Define Flexural Rigidity of Beams.

The product of young's modulus (E) and moment of inertia (I) is called Flexural Rigidity (EI) of Beams. The unit is $N mm^2$.

12. Define: Constant strength beam.

If the flexural Rigidity (EI) is constant over the uniform section, it is called Constant strength beam.

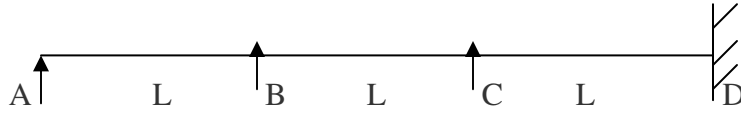
13. What is the sum of distribution factors at a joint?

Sum of distribution factors at a joint = 1.

14. Define the term 'sway'.

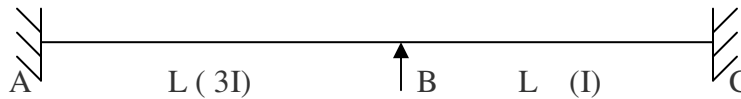
Sway is the lateral movement of joints in a portal frame due to the unsymmetry in dimensions, loads, moments of inertia, end conditions, etc.

15. Find the distribution factor for the given beam.



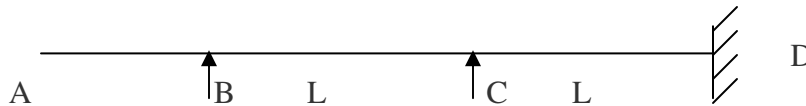
Joint	Member	Relative stiffness	Sum of Relative stiffness	Distribution factor
A	AB	$4EI/L$	$4EI/L$	$(4EI/L)/(4EI/L) = 1$
B	BA	$3EI/L$	$3EI/L + 4EI/L = 7EI/L$	$(3EI/L)/(7EI/L) = 3/7$
	BC	$4EI/L$		$(4EI/L)/(7EI/L) = 4/7$
C	CB	$4EI/L$	$4EI/L + 4EI/L = 8EI/L$	$(4EI/L)/(8EI/L) = 4/8$
	CD	$4EI/L$		$(4EI/L)/(8EI/L) = 4/8$
D	DC	$4EI/L$	$4EI/L$	$(4EI/L)/(4EI/L) = 1$

16. Find the distribution factor for the given beam.



Joint	Member	Relative stiffness	Sum of Relative stiffness	Distribution factor
A	AB	$4E(3I)/L$	$12EI/L$	$(12EI/L)/(12EI/L) = 1$
B	BA	$4E(3I)/L$	$12EI/L + 4EI/L = 16EI/L$	$(12EI/L)/(16EI/L) = 3/4$
	BC	$4EI/L$		$(4EI/L)/(16EI/L) = 1/4$
C	CB	$4EI/L$	$4EI/L$	$(4EI/L)/(4EI/L) = 1$

17. Find the distribution factor for the given beam.



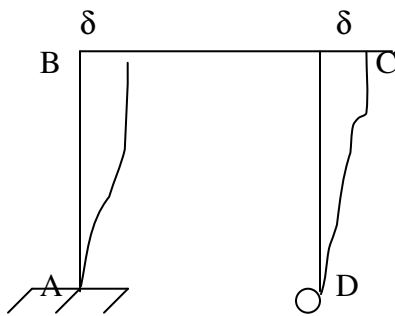
Joint	Member	Relative stiffness	Sum of Relative stiffness	Distribution factor
B	BA	0 (no support)	$3EI/L$	0
	BC	$3EI/L$		$(3EI/L)/(3EI/L) = 1$
C	CB	$3EI/L$	$3EI/L + 4EI/L = 7EI/L$	$(3EI/L)/(7EI/L) = 3/7$
	CD	$4EI/L$		$(4EI/L)/(7EI/L) = 4/7$
D	DC	$4EI/L$	$4EI/L$	$(4EI/L)/(4EI/L) = 1$

18. What are the situations where in sway will occur in portal frames?

- Eccentric or unsymmetric loading
- Unsymmetrical geometry
- Different end conditions of the columns
- Non-uniform section of the members
- Unsymmetrical settlement of supports
- A combination of the above

19. What is the ratio of sway moments at column heads when one end is fixed and the other end hinged? Assume that the length and M.I of both legs are equal.

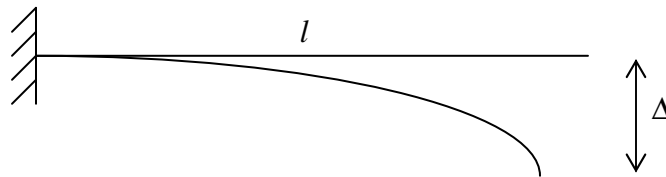
Assuming the frame to sway to the right by δ



Ratio of sway moments =

$$\frac{M_{BA}}{M_{CD}} = \frac{- \left[\frac{6EI\delta}{l^2} \right]}{- \left[\frac{3EI\delta}{l^2} \right]} = 2$$

20. A beam is fixed at its left end and simply supported at right. The right end sinks to a lower level by a distance ' Δ ' with respect to the left end. Find the magnitude and direction of the reaction at the right end if l is the beam length and EI , the flexural rigidity.



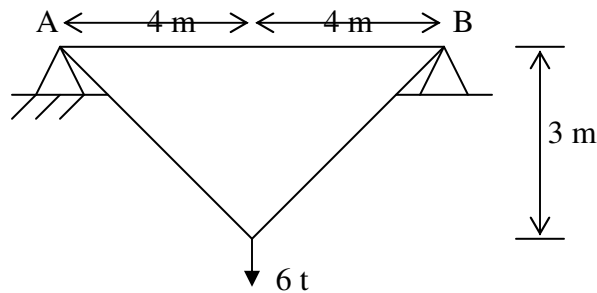
$$M_A \text{ (due to sinking of B)} = \frac{3EI\Delta}{l^2}$$

21. What are symmetric and antisymmetric quantities in structural behaviour?

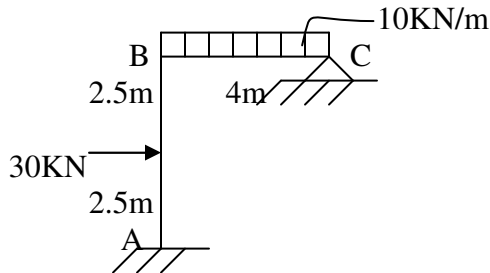
When a symmetrical structure is loaded with symmetrical loading, the bending moment and deflected shape will be symmetrical about the same axis. Bending moment and deflection are symmetrical quantities.

Part – B

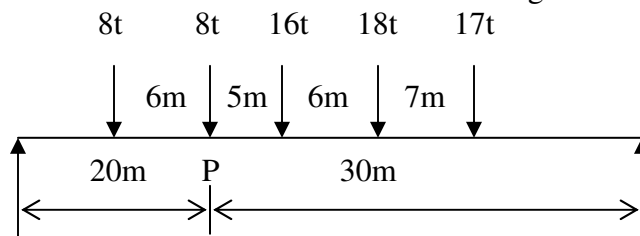
1. A pin-jointed frame shown in Fig. is carrying a load of 6 tonnes at C. Find the vertical as well as horizontal deflection at C. Take area of member AB as 10cm^2 and those of members AC and BC as 15cm^2 . $E = 2 \times 10^3 \text{ t/cm}^2$.



2. Using the method of virtual work, determine the horizontal displacement of a point C of the frame shown in Fig. Take $E = 2 \times 10^5 \text{ N/mm}^2$, $I = 4 \times 10^6 \text{ mm}^4$.

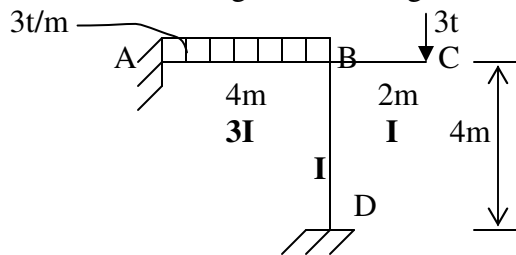


3. Using Influence line diagram, find (i) maximum Bending moment (ii) maximum positive and negative shears at 4m from left support A of a simply supported beam of span 10m, when a train of 4 wheel loads 10kN, 15kN, 30kN and 30kN spaced at 2m, 3m and 3m respectively cross the span with the 10kN load leading.
4. Draw the Influence line diagram for shear force and bending moment for a section at 5m from left support of a simply supported beam, 20m long. Hence calculate the maximum B.M and S.F at the section. Due to uniformly distributed rolling load of length 8m and intensity 10kN/m run.
5. For the span shown in Fig., obtain the bending moment at a section P, 20m from A, due to given loads in the position indicated. Also determine the position of the loads for maximum bending moment of section P and the value of maximum bending moment.

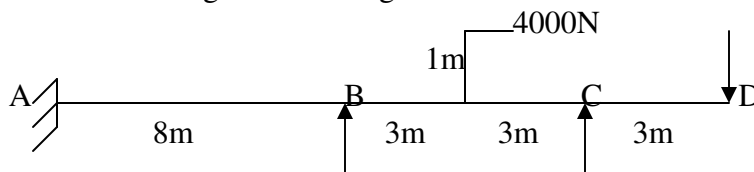


6. A uniformly distributed load of 5t/m , longer than span, rolls over a beam of 25m span. Using influence line, determine the maximum shear force and bending moment at a section 10m from the left end support.
7. A system of concentrated loads shown in Fig. Rolls from left to right across a beam simply supported over a span of 10m, the 10kN load leading. For a section 4m from the left support, determine maximum shear force and bending moment.
8. A udl of 40t/m covers left hand half of the span of a two hinged parabolic arch, span 36m and central rise 8m. Determine the position and magnitude of maximum bending moment. Also find shear force and normal thrust at the section. Assume that the moment of inertia at a section varies as secant of slope at the section.

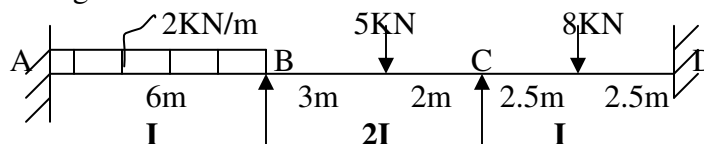
9. A three hinged parabolic arch of span 40m and rise 8m carries a udl of 30KN/m over the left half span. (i) Analyse the arch and draw the bending moment diagram. (ii) Also evaluate the thrust and shear force at a section 10m from left hinge.
10. A three hinged circular arch of span 16m and rise 4m is subjected to two point loads of 100KN and 80KN at left and right quarter span points respectively. Find the reaction at the supports. Find also BM, radial shear and normal thrust at 6m from the left support.
11. A three hinged parabolic arch of span 30m has its supports at depths of 4m and 16m below crown C. The arch carries a load of 80KN at a distance of 5m to the left of C and a second load of 100KN at 10m to the right of C. Determine the reactions at supports and BM under the loads.
12. A three hinged parabolic arch has a horizontal span of 30m with a central rise of 5m. a point load of 10KN moves across the span from left to right. Calculate the maximum positive and negative moments at a section 8m from the left hinge. Also, calculate the position and magnitude of the absolute maximum BM that may occur in the arch.
13. A three hinged parabolic arch has a horizontal span of 36m with a central rise of 6m. A point load of 8KN moves across from left to right. Calculate the maximum sagging and hogging BM at the section 9m from the left hinge. Calculate also the position and amount of absolute maximum BM that may occur on the arch.
14. A two hinged parabolic arch of span 25m, rise 6m is subjected to a udl of 15KN/m over the left half span and a point load of 25KN AT 9.5m from the right support. Find the support reactions, BM, radial shear and normal thrust at 4m from the left support.
15. A three hinged parabolic arch is subjected to a udl of 10KN/m for the left half portion. Using ILD, find the BM, radial shear and normal thrust at a section 4m from the left support.
16. A beam ABC supported on a column BD is loaded as shown in Fig. Analyse the frame by slope deflection method and draw bending moment diagram.



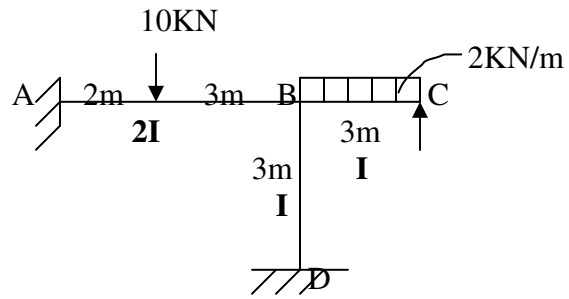
17. A continuous beam of constant moment of inertia is loaded as shown in Fig. Find the support moments and draw bending moment diagram.



18. Analyse the continuous beam shown in Fig. by moment distribution method and draw the bending moment diagram.



19. Analyse the frame shown in Fig. by moment distribution method and draw BMD.



20. Analyse the frame shown in problem.16 by moment distribution method.
21. Analyse the beam shown in problem.17 by moment distribution method
22. Analyse the frame shown in problem.18 by slope deflection method.
23. Analyse the beam shown in problem.19 by slope deflection method.

Answers

Refer Class Notes