VII Semester Mechanical Engineering

ME 1401 - FINITE ELEMENT ANALYSIS

Short Q & A

1) What is meant by finite element analysis?

Finite element method is a numerical method for solving problems of engineering mathematical physics.

In the finite element method, instead of solving the problem for the entire body in one operation, we formulate the equations for each finite element and combine them to obtain the solution of the whole body.

2) What is meant by finite element?

A small unit having definite shape of geometry and nodes is called finite element.

3) State the methods of engineering analysis.

There are three methods of engineering analysis. They are:

- 1. Experimental methods.
- 2. Analytical methods.
- 3. Numerical methods or approximate methods.
- 4) Give examples for the finite element.

1. One dimensional elements:	(a) Truss element			
	(b) Bar, Beam element			

2. Two dimensional elements (a) Triangular element

(b) Rectangular elements

- 3. Three dimensional elements(a) Tetrahedral elements.(b) Hexahedral elements.
- 5) What is meant by node or Joint?

Each kind of finite element has a specific structural shape and is interconnected with the adjacent elements by nodal points or nodes. At the nodes, degrees of freedom are located. The forces will act only at nodes and not at any other place in the element. 6) What do you mean by discretization?

Discretization is the basis of finite element method. The art of subdividing a structure into a convenient number of smaller components is known as discretization.

7) What are the types of boundary conditions?

There are two types of boundary conditions, they are

Primary boundary condition.

Secondary boundary condition.

8) What are the three phases of finite element method.

The three phases are 1. Preprocessing

2. Analysis

3.Postprocessing

9) What is structural and non-structural problem?

Structural problem: In structural problems, displacement at each nodal point is obtained. By using these displacement solutions, stress and strain in each element can be calculated.

Non Structural problem: In non structural problem, temperatures or fluid pressure at each nodal point is obtained. By using these values, Properties such as heat flow, fluid flow, etcfor each element can be calculated.

10) What are the methods are generally associated with the finite element analysis?

The following two methods are generally associated with the finite element analysis. They are

1. Force method.

- 2. Displacement or stiffness method
- 11) Explain force method and stiffness method?

In force method, internal forces are considered as the unknowns of the problem. In displacement or stiffness metho, displacement of the node are considered as the of the problem. Among them two approaches, displacement method is desirable.

12) What is polynomial type of interpolation functions are mostly used in FEM?

The polynomial type of interpolation functions are mostly used due to the following reasons:

- 1. It is easy to formulate and computerize the finite element equations.
- 2. It is easy to perform differentiation or integration.
- 3. The accuracy of the results can be improved by increasing the order of the polynomial .
- 13) Name the variational methods.

1.Ritz method.

2. Rayleigh – Ritz method

- 14) Name the weighted residual methods.
 - 1.Point collocation method.
 - 2. subdomain collocation method.
 - 3. Least square method
 - 4. galerkin's method
- 15) What is meant by post processing?

Analysis and evaluation of the solution results is referred to as post processing. Post processor computer programs help the user to interpret the results by displaying them in graphical form.

16) What is Rayleigh ritz method?

Rayleigh ritz method is a integral approach method which is useful for solving complex structural problems, encountered in finite element analysis. This method is possible only if a suitable functional is available.

17) What does assemblage mean?

The art of subdividing a structure into a convenient number of smaller components is known as discretization. These smaller components are then put together. The process of uniting the various elements together is called assemblage.

18) What is meant by DOF?

When the force or reaction acts at nodal point, node is subjected to deformation. The deformation includes displacement, rotations, and/or strains. These are collectively known as degrees of freedom (DOF).

19)What is aspect ratio?

Aspect ratio is defined as the ratio of the largest dimension of the element to the smallest dimension. In many cases, as the aspect ratio increases, the inaccuracy of the solution increases. The conclusion of many researches is that the aspect ratio should be close to unity as possible.

20) What is truss element?

The truss elements are the part of a truss structure linked together by point joints, which transmit only axial force to the element.

21) List the two advantages of post processing?

1.Required result can be obtained in graphical form.

2. Contour diagrams can be used to understand the solution easily and quickly.

22) If a displacement field in x direction ils given by $u=2x^2+4y^2+6xy$. Determine the strain in x direction.

 $U=2x^{2}+4y^{2}+6xy$

Strain, $e = \delta u / \delta x = 4x + 6y$

23) What are h and p versions of finite element method?

H version and p versions are used to improve the accuracy of the finite element method.

In h versions, the order of polynomial approximation for all elements is kept constant and the number of elements is increased.

In p version, the number of elements is maintained constant and the order of polynomial approximation of element is increased.

24) During discretization, mention the places where it is necessary to place a node

The following places are necessary to place a node during discretization process.

- 1. Concentrated load-acting point.
- 2. Cross section changing point
- 3. Different material interjunction point
- 4. Sudden change in load point.
- 25) What is the difference between static and dynamic analysis?

Static analysis: The solution of the problem does not vary with time is known as static analysis.

Example: Stress analysis on a beam.

Dynamic analysis: The solution of the problem varies with time is known as dynamic analysis.

Example: vibration analysis problems.

26) Name the four FEA softwares?

- 1. ANSYS
- 2. NASTRAN
- 3. COSMOS

4. NISA

27) Differentiate between global and local axes.

Local axes are established in an element. Since it is in the element level, they change with the change in orientation of the element. The direction differs from element to element.

Global axes are defined for the entire system. They are same in direction for all the elements even though the elements are differently oriented.

28) Distinguish between potential energy function and potential energy functional.

If a system has finite number of degrees of freedom ($q_1 q_2$ and q_3) then the potential energy is expressed as,

 $\pi = f(q_1 q_2 and q_3)$

It is known as function.

If a system has infinite degrees of freedom, then the potential energy is expressed as,

 $\pi = \int f(x, y, dy/dx, d^2y/dx^2, \dots) dx$

It is known as functional.

29) What are the types of loading acting on the structure? There are three types of loading acting on the body. They are:

1. Body force (f)

- 2. Traction force (T)
- 3. Point load (P)

30) Define body force (f).

A body force is distributed force acting on every elemental volume of the body.

Unit: Force per unit volume.

Example: Self-weight due to gravity

31) Define traction force (T)

Traction force is defined as a distributed force acting on the surcace of the body

Unit: force per unit area.

Examples: Frictional resistance, viscous drag, surface shear etc.

32) What is point load (P)

Point load is force acting at a particular point, which causes displacement.

33) What are the basic steps involved in the finite element modeling.

Finite element modeling consists of the following:

1.Discretization of structure

- 2. Numbering of nodes.
- 34) What is discretization?

The art of subdividing a structure into a convenient number of smaller componentsis known as descretization.

35) What is the classification of co-ordinates?

The co ordinates are generally classified as follows:

- 1. Global co-ordinates
- 2. Local co-ordinates
- 3. Natural co-ordinates
- 36) What is Global co-ordinates?

The points in the entire structure are defined using co-ordinate system is known as global co-ordinate system.



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1	2	3	4	5	6

37) What is natural co-ordinates?

A natural co-ordinate system is used too defined any point inside the element by a set of dimensionless numbers, whose magnitude never exceeds unity. This system is very useful in assembling of stiffness matrices.

38) Define shape function.

In finite element method, field variables within an element are generally expressed by the following approximate relation

 $\Phi(\mathbf{x}, \mathbf{y}) = \mathbf{N}_1(\mathbf{x}, \mathbf{y}) \ \phi_1 + \mathbf{N}_2(\mathbf{x}, \mathbf{y}) \ \phi_2 + \mathbf{N}_3(\mathbf{x}, \mathbf{y}) \ \phi_3$

Where ϕ_1, ϕ_2 and ϕ_3 are the values of the field variables at the nodes and N_1 , N_2 and N_3 are the

interpolation functions.

 $N_1\,,\!N_2$ and $N_3\,$ also called shape function because they are used to express the geometry or shape of the element.

39) What are the characteristics of shape function?

The characterstics of shape function are as follows:

1. The shape function has unit value at one nodal point and zero value at other nodal points.

The sum of shape

function is equal to one.

2.

40) Why polynomial are generally used as shape function?

Polynomials are generally used as shape function due to the following reasons.

- 1. Differentiation and integration of polynomial are quite easy.
- 2. The accuracy of the results can be improved by increasing the order of the polynomial.

3.

It is easy to formulate

and computerize the finite element equations.

41) How do you calculate the size of the global stiffness matrix?

Global stiffness matrix size = Number of nodes X degree of freedom per node

1)

Give th general

expression for element stiffness matrix.

Stiffness matrix $[k] = \int [B]^{T} [D] [B] dv$ [B] – Strain displacement matrix [row matrix]

[D] – Stress, Strain relationship matrix [Row matrix]

42) Write down the expression of stiffness matrix for one dimensional bar element.

Stiffness matrix [k] = AE $\begin{bmatrix} 1 & -T \\ -1 & 1 \end{bmatrix}$

A - Area of the bar element.

- E-Youngs modulus of bar element
- L lenth of the bar element.
- 43) State the properties of a stiffness matrix.

The properties of a stiffness matix [k] are

1.

2. any column must be equal to zero.

3. So, the determinant is equal to zero.

44) Write down the general finite element equation.

General finite element equation is,

 $\{F\} = [K] \{ \cup \}$

{F} - Force vector [column matrix]

[k] - Stiffness matrix [row matix]

{u} - Degrees of freedom [coloumn matrix]

It is symmetric matrix.

The sum of elements in

It is an unstable element.

45) Write down the finite element equation for one dimensional two noded bar element.

The finite element equation for one-dimensional two noded bar element is,



46) What is truss?

A truss is defined as a structure, made up of several bars, riveted or welded together.

47) State the assumptions are made while finding the forces in a truss.

The following assumptions are made while finding the forces in a truss.

- 1. All the members are pin jointed.
- 2. The truss is loaded only at the joints.
- 3. The self-wight of the members is neglected unless stated.

48) Write down the expression of stiffness matrix for a truss element.

A – Area

E – Youngs modulus

 l_e - Length of the element

l,m – Direction cosines

49) Write down the expression of shape function N and displacement u for onedimensional bar element.

For one dimensional bar element

Displacement function, $u = N_1 u_1 + N_2 U_2$

Where, shape function $N_1 = I-x / I$

shape function N₂= x/l

50) Define total potential energy.

The total potential energy π of an elastic body, is defined as the sum of total strain energy U and potential energy of the external forces,(W).

Total potential energy, π = Strain energy (U) + Potential energy of the external forces (W).

51) State the principle of minimum potential energy.

The principle of minimum potential energy states: Among all the displacement equations that satisfy internal compatibility and the boundary conditions, those that also satisfy the equations of equilibrium make the potential energy a minimum in a stable system.

52) What is the stationary property of total potential energy?

If a body is in equilibrium, its total potential energy π is stationary.

For stable equilibrium, $\delta^2 \pi > 0$, other wise π is minimum for stable equilibrium.

For neutral equilibrium, $\delta^2 \pi = 0$. In this case π is unchanging.

For unstable equilibrium, $\delta^2 \pi < 0$, other wise π is maximum.

53) State the principle of virtual work?

A body is in equilibrium if the internal virtual work equals the external virtual work for every kinematical admissible displacement field.

54) Distinguish between essential boundary conditions and natural boundary conditions.

There are two types of boundary conditions. They are:

1. Primary boundary condition

The boundary condition, which in terms of field variable, is known as primary boundary condition.

Secondary boundary conditions:

2.

The boundary conditions, which are in the differential form of field variables, are known as secondary boundary condition.

Example: A bar is subjected to axial load as shown in fig.



In this problem, displacement u at node 1 = 0, that is primary boundary condition. EA du/dx = P, that is secondary boundary condition.

55) What are differences between boundary value problem and initial value problem?

The solution of differential equation is obtained for physical problems, which satisfies some specified conditions known as boundary conditions.

The differential equation together with these boundary conditions, subjected to a boundary value problem.

The differential equation together with initial conditions subjected to an initial value problem.

Examples: Boundary value problem.

 $d^{2}y/dx^{2} - a(x) dy/dx - b(x)y - c(x) = 0$

with boundary conditions, y(m) = a and

$$y(n) = \beta$$

initia I value problem, $ax^2 + bx + c = 0$

Boundary conditions: x(0) = 0

X(0) = 7

56) How do you define two-dimensional elements?

Two dimensional elements are defined by three or nodes in a two dimensional plane (ie x, y plane). The basic element useful for two dimensional analysis is the triangular element.

57) What is CST element?

Three-nodded triangular element is known Constant Strain Triangle (CST) which is shown in fig. it has six unknown displacement degrees of free (u1 v1 u2 v2, u3 v3). The element is called CST because it has a constant strain through it.



1 U₁

Merit:

Calculation of stiffness matrix is easier. Demerit:

The strain variation with in the element is considered as constant. So, the results

will be poor.

58) What is LST element?

Six noded triangular elements are known as linear strain triangle (LST), which is shown in fig. It has twelve unknown displacement degrees of freedom. The displacement functions for the element are quadratic instead of linear as in the CST.



4

59) What is QST element?

Ten noded triangular elements is known as quadratic strain triangle (QST), which is shown if, fig. it is also called cubic displacement triangle.

2



60) What is meant by plane stress analysis?

1

Plane stress is defined to be a state in which the normal stress (σ) and shear stress directed perpendicular to the plane is assumed to be zero.

61) Define plane strain analysis

Plane strain is defined to be a state of strain in which the strain normal to the xy plane and the shear strains are assumed to be zero.

Where N_1 , N_2 , N_3 are shape functions.

63) Write a strain-displacement matrix for CST element.

Strain displacement matrix for CST element is

$$[B] = \frac{1}{2A} \begin{pmatrix} Q_1 & 0 & q_2 & 0 & q_3 & 0 \\ 0 & r_1 & 0 & r_2 & 0 & r_3 \end{pmatrix}$$

 $- R_1 q_1 r_2 q_2 - r_3 q_3$

Where A = Area of the element

 $\begin{array}{cccc} Q_1 = y_2 - y_3 & q_2 = y_3 - y_1 & q_3 = y_1 - y_2 \\ R_1 = x_3 - x_2 & r_2 = x_1 - x_3 & r_3 = x_2 - x_1 \end{array}$

64) Write down the stress strain relationship matrix for plane strain condition.

For plane strain problems, stress strain relationship matrix is,

$$\begin{bmatrix} D \end{bmatrix} = \underbrace{E \begin{bmatrix} (1-V) & V & 0 \\ 0 & (1-V) & 0 \\ (1+V)(1-2V) & 0 & 0 \\ \end{bmatrix} = \underbrace{E \begin{bmatrix} 0 & (1-V) & 0 \\ 0 & 1-2V/2 \\ \end{bmatrix}$$

65) Write down the stiffness matrix equation for two-dimensional CST element.

Stiffness matrix, $[k] = [B]^{T} [D] [B] A t$

[B] = Strain displacement matrix

[D] = Stress strain matrix

A = Area of the element

T = Thickness of the element

66) Write down the stress strain relation ship matrix for plane stress condition.

For plane stress problem, stress strain relation ship matrix is

$$[D] = E \begin{bmatrix} & 1-v & v & 0\\ 0 & 1-v & 0\\ (1+v)(1-2v) & 0 & 0 & 1-2v/2 \end{bmatrix}$$

67) Write down the expression for the shape function for a constant strain triangular element.

For CST element,

Shape function, $N_1 = P_1 + q_1x + r_1y$

$$N_{2} = p_{2}+q_{2}x+r_{2}y$$

$$2A$$

$$N_{3} = p_{3}+q_{3}x+r_{3}y$$

$$2A$$
Where $P_{1} = x_{2}y_{3} - x_{3}y_{2}$

$$P_{2} = x_{3}y_{1} - x_{1}y_{3}$$

$$P_{3} = x_{1}y_{2} - x_{2}y_{1}$$

$$q_{1} = y_{2} - y_{3}$$

$$q_{2} = y_{3} - y_{1}$$

$$q_{3} = y_{1} - y_{2}$$

$$r_{1} = x_{3} - x_{2}$$

$$r_{2} = x_{1} - x_{3}$$

$$r_{3} = x_{2} - x_{1}$$

68) What is axisymmetric element?

Many three dimensional problems in engineering exhibit symmetry about an axis of rotation. Such types of problems are solved by a special two-dimensional element called the axisymmetric element.

69) What are the conditions for a problem to axisymmetric?

- 1. The problem domain must be symmetric about the axis of revolution.
- 2. All boundary conditions must be symmetric about the axis of revolution.
- 3. All loading condition must be symmetric about the axis of revolution.
- 70) Write down the displacement equation for an axisymmetric triangular element.



71) Write down the shape function for an axisymmetric triangular element.

$$N_{1} = \alpha_{1} x + \beta_{1} y + \Box_{1} z$$

$$2A$$

$$N_{2} = \alpha_{2} x + \beta_{2} y + \Box_{2} z$$

$$2A$$

$$N_{3} = \alpha_{3} x + \beta_{3} y + \Box_{3} z$$

$$2A$$
Where $\alpha_{1} = r_{2} z_{3} - r_{3} z_{2}$

$$\alpha_{2} = r_{3} z_{1} - r_{1} z_{3}$$

$$\alpha_{3} = r_{1} z_{2} - r_{2} z_{1}$$

$$\beta_{1} = y_{2} - y_{3}$$

 $\begin{array}{c} \beta_{3}=z_{1}-z_{2}\\ \Box_{1}=r_{3}-r_{2}\\ \end{array} \qquad \Box_{2}=r_{1}-r_{3}\\ \end{array} \qquad \Box_{3}=r_{2}-r_{1}\\ \end{array}$ 72) Give the strain-displacement matrix equation for an axisymmetric triangular element.

Strain-displacement matrix,



73) Write down the stress - strain relation ship matrix for an axisymmetric triangular element.



	E		1-🗆		0
Stress- strain relationship matrix,[D] 0	(1+ □)(1-2 □)				1- 🗆
□/2	_	0	0	0	1-2

- E youngs modulus
- V poisson ratio

74) Give the stiffness matrix equation for an axisymmetric triangular element.

Stiffness matrix, $[k] = 2\pi r A [B]^{T} [D] [B]$

Where, co-ordinate $r = r_1 + r_2 + r_3 /3$

A – area of the triangular element.

75) What are the ways in which a three dimensional problem can be reduced to a two dimensional approach.

1. Plane stress: One dimension is too small when compared to other two dimensions.

Example: gear – thickness is small

- 2. Plane strain: one dimension is too large when compard to other two dimensions example : Long pipe [length is long compared to diameter]
- 3. Axisymmetric : geometry is symmetric about the axis.

Example: cooling tower

76) Calculate the jacobian of the transformation J for the triangular element shown if fig.





J = 3+6 = 9 units

77) What is the purpose of isoparametric elements?

It is difficult to represent the curved boundaries by straight edges finite elements. A large number of finite elements may be used to obtain reasonable resemblance between original body and assemblage. In order to overcome this drawback, isoparametric elements are used i.e. for problems involving curved boundaries; a family of elements known as "isoparametic elements" is used

78) Write down the shape function for 4 noded rectangular elements using natural co-ordinate system.

Shape functions: $N_1 = \frac{1}{4} (1-\varepsilon) (1-\eta)$

 $N_2 = \frac{1}{4} (1+\varepsilon) (1-\eta)$

$$N_3 = \frac{1}{4} (1+\varepsilon) (1+\eta)$$

$$N_4 = \frac{1}{4} (1-\varepsilon) (1+\eta)$$

Where, ε and η are natural co-ordinates.

79) Write down the jacobian matrix for four noded quadrilateral element.

Jacobian matrix,
$$\begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} J_1 \\ J_{21} \end{bmatrix} = \begin{bmatrix} J_{12} \end{bmatrix}$$

Where, $J_{11} = \frac{1}{4}[-(1-\eta)x_1 + (1-\eta)x_2 + (1+\eta)x_3 - (1+\eta)x_4]$

$$J_{12} = \frac{1}{4}[-(1-\eta)y_1 + (1-\eta)y_2 + (1+\eta)y_3 - (1+\eta)y_4]$$

$$J_{21} = \frac{1}{4}[-(1-\varepsilon)x_1 - (1+\varepsilon)x_2 + (1+\varepsilon)x_3 + (1-\varepsilon)x_4]$$

$$J_{22} = \frac{1}{4}[-(1-\varepsilon)y_1 - (1+\varepsilon)y_2 + (1+\varepsilon)y_3 + (1-\varepsilon)y_4]$$

Where, ε and η are natural co-ordinates.

 $x_1 x_2 x_3 x_4 y_1 y_2 y_3 y_4$ are Cartesian coordinates.

80) Write down the stiffness matrix equation for four noded isoparametric quadrilateral elements.

-1 -1

Where t = thickness of the element

|J|=Determinant of the jacobian

 ε and η are natural co-ordinates

[B] Strain displacement matrix

[D] Stress strain relationship matrix

81) Write down the element force vector equation for four noded quadrilateral elements.

Force vector, $\{F\} = [N]^{T} \left\{ \begin{array}{c} F_{x} \\ F_{y} \end{array} \right\}$

N is the shape function

 F_x is the load or force in x direction

 F_y is the force on y direction

82) Write down the Gaussian quadrature expression for numerical integration.

Gaussian quadrature expression

 $\int f(x) dx = \sum w_i f(x_i)$

Wi weight function

F(x_i) values of the function at pre-determined sampling points.

83) Define super parametric element.

If the number nodes used for defining the geometry is more than number of nodes used for defining the displacements is known as super parametric element.

84) What is meant by sub parametric element?

If the number of nodes used for defining the geometry is less than number of nodes used for defining the desplacements known as isoparametric element.

85) What is meant by iso parametric element?

If the number of nodes used for defining the geometry is same as number of nodes used for defining the displacements is know as isoparametric element.

86) Is beam element an isoparametric element?

Beam element is not an isoparametric element since the geometry and displacements are defined by different order interpolation functions.

87) What is the difference between natural co-ordinates and simple natural co-ordinate?

A natural co-ordinate is one whose value lies between zero and one.

Examples: $L_2 = x/I$; I = (1-x/I)

Area co-ordinates : $L_1 = A_1/A$; $L_2 = A_2/A$; $L_3 = A_3/A$

A simple natural co ordinates is one whose value lies between -1 to +1

88) Give examples for essential (forced or geometric) and non-essential (natural) boundary conditions.

The geometric boundary conditions are displacement, slopes, etc. the natural boundary conditions are bending moment, shear force, etc.

Descriptive type

89) Explain the general steps in FEA with the help of a flowchart?

90) A beam AB of span 'L' simply supported at ends and carrying a concentrated load 'W' at the center 'C'. Determine the deflection at midspan by using the Rayleigh-Ritz method.

91) Solve the equations using Gauss- Elimination method

$$2x + 4y + 2z = 152x + y + 2z = -54x + y - 2z = 0$$

92) Describe the four types of weighted residual method.

93) Derive the stiffness matrix [K] for the truss element

94) Derive the shape function for one-dimensional bar element.

95) Using finite element, find the stress distribution in a uniformly tapering bar of circular cross sectional area 3cm^2 and 2 cm^2 at their ends, length 100mm, subjected to an axial tensile load of 50 N at smaller end and fixed at larger end. Take the value of Youngs modulus as $2 \times 10^5 \text{N/mm}^2$.

96) (i) Explain the Galerkin's method.

(ii) Explain the Gaussian elimination.

97) Derive the constitutive matrix for 2D element.

98) Derive the expression for the stiffness matrix for an axisymmetric shell element.

99) Explain the terms plane stress and plane strain conditions. Give the constitutive laws for these cases.

100) Derive the element stiffness matrix for a linear isoparametric quadrilateral element.

101) Evaluate the integral by using Gaussian quadrature $\int x^2 dx$.
