MIT ARTS COMMERCE AND SCIENCE COLLEGE, ALANDI (D), PUNE-412105

QUESTION BANK : COMPUTATIONAL GEOMETRY

MATHEMATICS PAPER-I SYBSC(SEM-II)

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Q.I) Questions 1 to 40 carries 1mark each:

1) What will be the effect, if in combined transformation the order of the transformation is changed?
2) Determine whether the transformation matrix $[T]=\begin{bmatrix} \cos\theta & \sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ represents a solid body Transformation, for any real number θ?
3) If a square with sides 2 cms is reflected through y axis, then what is the area of transformed figure?
4) Write the transformation matrix required to create bottom view of the object.
5) Write the transformation matrix to shear in x direction proportional to y coordinate by a factor 5 And proportional to z coordinate by a factor 6.
6) What is the point at infinity on the $y-axis$ in the positive direction?
7) Explain the difference between affine and perspective transformations.
8) Write the transformation matrix for rotation about $y-axis$ through an angle $=\frac{\pi^c}{2}$.
9) Find the angle $\delta\theta$, to generate 5 points on the hyperbolic segment in the first quadrant for $6 \le x \le 12$, where the parametric equations of the hyperbola are $x=3\cosh\theta \,\&\, y=2\sinh\theta$.
10) Mention any two applications of space curves.
11) The circle of area 10 cm^2 is scaled uniformly by factor 2, and then what is the area of the transformed figure?
12) Explain the term: Point at infinity.
13) Write a 2D transformation matrix for overall scaling by factor s. What is its effect if 0 < s < 1?
14) A shadow of a person standing on ground is formed by sunlight. What type of projection is this?
15) Write the transformation matrix which is required to transform the plane x = 0 to the plane x = 5.

16) Let L be a line with d.r.s 1, 1, 1. Find the angle α, if L is rotated about X axis by angle α and then rotated about Y axis by angle β, so that L coincides with Z axis.

17) Find the angle $\delta\theta$ to generate 8 equidistant points on an elliptical arc in the 1st and 2nd quadrant for $\frac{x^2}{4}+\frac{y^2}{16}=1$.

18) Write the matrix for cabinet projection if the horizontal inclination angle $\alpha = 25^{\circ}$.

19) Write any two properties of Bezier Curve.

20) What is the transformation represented by the matrix $[T]$?

$$[T]=\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & -0.5 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

21) What is the effect of the transformation matrix $[T]=\begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix}$ on the 2 – dimensional object?

22) If the circle of circumference 14π is uniformly scaled by 3 units, what is the area of the transformed circle.

23) Find the angle through which the line $y=-x$ rotated so that it is coincident with $x-axis$.

24) What is an apparent translation? How to obtain pure scaling without an apparent translation?

25) Write the transformation matrix for shear in x coordinate by a factor of 2 units proportional to z coordinate and shear in z coordinate by a factor of 3 units proportional to x coordinate.

26) If $z=-5$ is the given plane, find the transformation matrix which when applied on the given plane, transforms the plane to $z=0$ plane.

27) Give two different aspects of perspective views experienced by human eye.

28) If 8 distinct uniformly spaced points on the periphery of an ellipse $\frac{x^2}{4}+\frac{y^2}{1}=1$ are to be generated and out of which three points $\{A[2\ 0], B[1.4\ 0.7], C[0\ 1]\}$ are given, then generate remaining points on the periphery, by using reflection.

29) Write the matrix equation form of a parametric equation of a Bezier curve for 3 control points B_0, B_1, B_2.

30) Let $[x]$ represent n points of the circle $x^2 + y^2 = 1$ and $\left[x^*\right]$ represent n points of the circle $(x+3)^2 + (y-2)^2 = 16$ where $\left[x^*\right] = [x][T_1][T_2]$. Write the transformation matrices $[T_1]$ and $[T_2]$.

31) What is the effect of the transformation matrix $[T] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$ on the 2 – dimensional object?

32) What is the determinant of the inverse of any pure rotation matrix?

33) If line L is transformed to the line L^* using a transformation matrix $[T] = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and slope of L^* is $\frac{2}{3}$, find the slope of the line L.

34) Write the rotation matrix required to rotate the line y = 2x so that it is coincident with x axis.

35) If y = 0 is the given plane. Find the transformation matrix which when applied on the given plane, transforms the plane to y = -2 plane.

36) Write the transformation matrix for orthographic projection to create the top view of the object.

37) Determine the foreshortening factors f_x and f_y if the transformation matrix for axonometric projection is given by $[T] = \begin{bmatrix} 0.99 & 0 & 0 & 0 \\ -0.09 & -0.66 & 0 & 0 \\ 0.08 & -0.74 & 0 & 0 \\ -2.5 & 3.05 & 0 & 1 \end{bmatrix}$.

38) Find an angle $\delta\theta$ to generate uniformly spaced 5 points on the circumference of a circle in the 2^{nd} and 3^{rd} quadrant.

39) State whether the following statement is true or false. Justify. There is no unique parametric representation of a circle.

40) Explain, what you mean by variation diminishing property of a Bezier curve.

Q.II] Questions 41 Onwards carries 5 marks each:

1) Explain different possible effects due to the entries of a general 2×2 transformation matrix $[T]=\begin{bmatrix} a & b \\ c & d \end{bmatrix}$.

2) Show that the parallel lines AB and CD are not transformed onto parallel lines, under transformation matrix, $[T]=\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$ where $A[1\ 2], B[2\ 4], C[2\ 6]\ \&\ D[3\ 8]$.

3) Rotate ΔABC about its centroid through an angle 45°, where $A[2\ -4], B[3\ 0]\ \&\ C[-2\ 1]$.

4) Write an algorithm for reflection through any arbitrary plane in space.

5) Find the combined transformation matrix for the following sequence of transformations: Translation in $x, y\ \&\ z$ directions by -1, 2 and 1 units respectively. Followed by scaling in $x\ \&\ y$ directions by factors 3 and ½ respectively. Followed by a reflection through the $yz-plane$. Apply it on the point $[1\ 3\ 2]$.

6) Consider the Bezier curve determined by the control points B_0 [4 3], B_1 [0 1] and B_2 [2 -1]. Find the first and the second derivatives of the curve at t = 0.3.

7) Let [X] be a square with vertices A, B, C, D, where A=[0 0], B=[1 0], C[1 1], and D=[0 1].Let [X'] be a quadrilateral with vertices A', B', C', D', where A' = [2 1], B' = [4 1], C' = [5 3] and D' = [3 3]. Find the 3 x 3 transformation matrix which transforms [X] to [X'], if overall scaling and projection are not applied.

8) Prove that, if a 2 x 2 transformation matrix is applied on a pair of parallel lines then they are transformed to a pair of parallel lines.

9) If an object [X] is reflected through the plane Z =3, then find the transformed object, where $[X]=\begin{bmatrix} 2 & 3 & 4 \\ 4 & -5 & 1 \end{bmatrix}$, using concatenated transformation matrix.

10) If the 2x2 transformation matrix transforms the point P and Q to the points P* and Q* respectively, then show that the same transformation transforms the midpoint of line segment PQ to the midpoint of line segment P*Q*.

11) Find the concatenated transformation matrix for the following transformation in order.
   a) Translate in X, Y, Z direction by -2, -2, -2 units respectively.
   b) Rotate about x-axis by an angle 45°.
   c) Reduce to half of its size.

12) Generate uniformly spaced 3 points on the parabolic segment in first quadrant for $3 \le x \le 12$ and equation of the parabola is $y^2 = 12x$.

13) If a 2x2 transformation matrix $[T]=\begin{bmatrix} 1 & 3 \\ -2 & 2 \end{bmatrix}$ is used to transform the line passing through two points A (3, -1/2) and B (0, 1), find equation of the resulting line.

14) If B_0 [2, 1], B_1 [4, 4], B_2 [5, 3], B_3 [5, 1] are vertices of a Be'zier polygon, then determine the point [P (0, 7)] of the Be'zier curve. Also find the first derivative of [P (t)] corresponding to t = 0.3.

15) Obtain the recurrence relation to generate uniformly spaced 9 points on the hyperbolic segment in the first quadrant for $9 \le x \le 18$ where equation of the hyperbola is $\frac{x^2}{9}-\frac{y^2}{25}=1$.

16) Find the cavalier projection with $\alpha = 30^\circ$ and cabinet projection with $\alpha = 25^\circ$ of the object represented by the following matrix.

$$[X]=\begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix}$$

17) Write the transformation matrix for the diametric projection with $f_z = 1/3$ and also find the foreshortening factors along x and y direction. $[Take\ \phi > 0,\ \theta < 0]$.

18) Consider a triangle with vertices {A [3 6], B [6 9], C [3 9]}. Rotate the triangle about a point (-2, 1) through an angle 35°. Write the position vectors of the transformed triangle.

19) Derive the transformation matrix for rotation about origin through an angle θ.

20) Obtain the concatenated matrix for the following sequence of transformations. First translation in x, y and z direction by -1, 2, 1 units respectively, followed by a rotation about z-axis by 90°, followed by a reflection in z=0 plane. Apply it on the point [1 2 3].

21) Generate uniformly spaced 3 points of the parabolic segment $y^2 = 8x$, in the first quadrant for $4 \le y \le 20$.

22) Write an algorithm for rotation about an arbitrary line l, passing through $[x_0, y_0, z_0]$ and having direction cosines C_x, C_y, C_z.

23) Determine if the transformation matrix $[T]=\begin{bmatrix} \frac{1}{\sqrt{10}} & \frac{3}{\sqrt{10}} \\ \frac{-3}{\sqrt{10}} & \frac{1}{\sqrt{10}} \end{bmatrix}$ preserves the length of the line segment and the angle between two intersecting lines. Justify your answer.

24) Generate uniformly spaced 6 points on the circle $x^2 + y^2 = 16$.

25) Write an algorithm to generate n points on hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, in the first quadrant for $x_{min} \le x \le x_{max}$.

26) Find the cavalier and cabinet projection of the line segment joining A [1 0 1 1] and B [0 1 1 1] with a horizontal inclination angle $\alpha = 30^\circ$.

27) Define foreshortening factors. Find the angles θ and ϕ when an isometric projection is formed by the rotation about Y axis through an angle ϕ, followed by the rotation about x axis through an angle θ and then orthographic projection on Z = 0 plane. How many isometric projections of any object are possible?

28) Generate uniformly spaced 7 points, in the 1st quadrant on an ellipse with equation $\frac{x^2}{25} + \frac{y^2}{16} = 1$.