



KINGS

COLLEGE OF ENGINEERING



DEPARTMENT OF ELECTRICAL AND ELECTRONICS ENGINEERING

QUESTION BANK

NAME OF THE SUBJECT: EE 1354 MODERN CONTROL SYSTEMS

YEAR / SEM : III / VI

UNIT – I

STATE SPACE ANALYSIS OF CONTINUOUS TIME SYSTEMS

PART – A (2 MARKS)

1. What are the drawbacks in transfer function model analysis?

- (i) Transfer function is defined under zero initial conditions.
- (ii) Transfer function is applicable to linear time invariant systems.
- (iii) Transfer function analysis is restricted to SISO systems.
- (IV) It does not provide information regarding the internal state of the system.

2. What is State and state variable?

The state is the condition of a system at any time instant, t . A set of variables which describes the state of the system at any time instant are called state variables.

3. What is a state vector ?

The state vector is a $(n \times 1)$ column matrix (or vector) whose elements are state variables of the system, (where n is the order of the system). It is denoted by $X(t)$.

4. What are the advantages of state space analysis?

- (i) The state space analysis is applicable to any type of systems. They can be used for modeling and analysis of linear and nonlinear systems, time invariant & time variant systems and Multiple input & Multiple output systems..
- (ii) The state space analysis can be performed with initial conditions.
- (iii) The variables used to represent the system can be any variables in the system.

(iv) Using this analysis the internal states of the system at any time instant can be predicted.

5. Write the state model of nth order system?

The state model of a system consists of state equation and output equation. The state model of a nth order system with m-inputs and p-outputs are

$$X(t) = AX(t) + BU(t) \quad \text{----- State equation}$$

$$Y(t) = CX(t) + DU(t) \quad \text{----- Output equation}$$

Where $X(t)$ = state vector of order (nx1)

$U(t)$ = Input vector of order (mx1)

A = system matrix of order (nxn)

B = Input matrix of order (nxm)

$Y(t)$ = Output vector of order (px1)

C = Output matrix of order (pxn)

D = transmission matrix of order (pxm)

6. The state model of a linear time invariant system is given by

$$X(t) = AX(t) + BU(t)$$

$Y(t) = CX(t) + DU(t)$. Write the expression for transfer function of the system.

$$Y(s)/U(s) = C(sI - A)^{-1}BU(s) + D$$

7. What is State diagram?

The Pictorial representation of the state model of the system is called state diagram. The state diagram of the system can be either in block diagram or signal flow graph form.

8. What are the advantages of state space modeling using physical variable?

- (i) The state variable can be utilized for the purpose of feedback.
- (ii) The solution of state equation gives time variation of variables which have direct relevance to the Physical system.

9. What are phase variables?

The phase variables are defined as those particular state variables which are obtained from one of the system variable and its derivatives. Usually the variable used is the system output and the remaining state variables and then derivatives.

10. Write the properties of state transition matrix.

(i) $\Phi(0) = e^{A \times 0} = I$ (Unit matrix)

(ii) $\Phi(t) = e^{At} = (e^{-At})^{-1} = [\Phi(-t)]^{-1}$

(iii) $\Phi(t_1 + t_2) = e^{A(t_1 + t_2)} = e^{At_1} e^{At_2} = \Phi(t_1) \Phi(t_2) = \Phi(t_2) \Phi(t_1)$

11. Write the solution of homogeneous state equations.?

The solution of homogeneous state equation is, $X(t) = e^{At} X_0$

Where , $X(t)$ = state vector at time , t

e^{At} =State transition matrix.

And X_0 =Initial condition vector at $t=0$

12. Write the solution of non- homogeneous state equations.?

The solution of non-homogeneous state equation is

$$X(t) = e^{A(t-t_0)} X(t_0) + \int e^{A(t-\tau)} B.U(\tau) d\tau$$

13. Write any two properties of eigenvalues.

(i) A matrix and its transpose have the same eigen values.

(ii) The product of the eigen values (counting multiplicities) of the matrix equals the determinant of the matrix.

14. What is similarity transformation?

The Process of transforming a square matrix A to another similar matrix B by a transformation $P^{-1}AP = B$ is called similarity transformation. The matrix P is called transformation matrix.

15. Define controllability and observability.

A system is said to be completely state controllable if it is possible to transfer the system state from any initial state $X(t_0)$ at any other desired state $X(t)$, in specified finite time by a control vector $U(t)$.

A system is said to be completely state observable if every state $X(t)$ can be completely identified by measurements of the output $Y(t)$ over a finite time interval.

16. What is pole placement by state feed back?

The pole placement by state feedback is a control system design technique ,in which the state variables are used for feedback to achieve the desired closed loop poles.

17. What is state observer?

A device (or a computer program) that estimates or observes the state variables is called state observer

18.What is the need for state observer?

In certain systems the state variables may not be available for measurement and feedback. In such situations we need to estimate the un measurable state variables from the knowledge of input and output. Hence a state observer is employed which estimates the state variables from the input and output of the system. The estimated state variable can be used for feedback to design the system by pole placement.

19.What is canonical form of state model?

If the system matrix, A is in the form of diagonal matrix then the state model is called canonical form.

20.What is meant by diagonalization?

The process of converting the system matrix A in to a diagonal matrix by a similarity transformation using the modal matrix M is called diagonalization.

PART – B

1. (a) Define controllability and observability. Explain both of them with the help of Kalman's test. (8)
 (b) Determine controllability and observability of the system described by

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$

$$Y = \begin{bmatrix} 4 & 5 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \quad (8)$$

2. (a) Derive the solution of homogeneous state equations. (8)

(b) A linear dynamical time invariant system represented by

$$\dot{x} = Ax + Bu \quad \text{where } A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -2 & -3 \end{bmatrix} \quad B = \begin{bmatrix} 0 & 1 \\ 0 & 0 \\ 1 & 0 \end{bmatrix}$$

Find if the system is completely controllable. (8)

3. (a) Construct a state model for a system characterized by the differential equation.

$$\frac{d^3 y}{dt^3} + 6 \frac{d^2 y}{dt^2} + 11 \frac{dy}{dt} + 6y + 4 = 0$$

Give the block diagram representation of the state model. (8)

(b) What are the advantages and disadvantages of state space analysis? and Define (i) Eigen values (ii) Eigen vectors (iii) state of a system (8)

4. (a) List the advantages of state variable method. Find the eigen values and

eigen vectors of given A matrix $A = \begin{bmatrix} 0 & 0 & 10 \\ 0 & 1 & 52 \\ -3 & -7 & 4 \end{bmatrix}$ (8)

(b) The state model of a system is given by

$$\dot{x} = Ax + Bu, \quad y = Cx$$

where $A = \begin{bmatrix} 0 & 0 & 1 \\ -2 & -3 & 0 \\ 0 & 2 & -3 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix}, \quad C = [1 \quad 0 \quad 0]$

Convert the state model to controllable phase variable form. (8)

5. Develop the state model of armature controlled and field controlled DC shunt motor also draw the state diagram. (16)

6. (a) Construct a state model for a system characterized by a differential equation

$$\overset{\dots}{y} + 6 \overset{\ddot{}}{y} + 11 \overset{\dot{}}{y} + 6y = \overset{\dots}{u} + 8 \overset{\ddot{}}{u} + 17 \overset{\dot{}}{u} + 8u$$
 (8)

(b) Derive the solution of Non-homogeneous state equations. (8)

7. (a) Obtain the transfer function for the following state model.

$$\dot{x} = \begin{bmatrix} -2 & 1 \\ 1 & -2 \end{bmatrix} X + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u \quad y = [0 \quad 1] X$$
 (8)

(b) Consider a linear system described by the differential equation $\overset{\ddot{}}{y} + 2 \overset{\dot{}}{y} + y = \overset{\dot{}}{u} + u$ with $x_1 = y$ & $x_2 = \dot{y} - u$ Determine the controllability and observability of the system. (8)

8. (a) Find $x_1(t)$ and $x_2(t)$ of the system described by

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -3 & -2 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$$

Where the initial conditions are

$$\begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix} \quad (8)$$

(b) Obtain the state model of the system whose transfer function is given as

$$\frac{s^2 + 4s + 3}{s^3 + 9s + 20} \quad (8)$$

9. (a) Consider a system described by the state equation: (time variant)

$$\dot{X}(t) = A(t)X(t) + bu(t) \text{ where } A(t) = \begin{bmatrix} 1 & e^{-t} \\ 0 & -1 \end{bmatrix}; \mathbf{b} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Is this system controllable at $t=0$? If yes, find the minimum-energy control to drive it

from $x(0)=0$ to $X' = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ at $t=1$ (8)

(b) Explain the pole placement design of continuous time system with a suitable example. (8)

10. Find the state model for the following transfer function

$$(a) g(s) = \frac{Y(s)}{U(s)} = \frac{3s^2 + 7s + 15}{s^3 + 7s^2 + 14s + 8} \quad (8)$$

$$(b) g(s) = \frac{Y(s)}{U(s)} = \frac{2s^3 + 10s^2 + 21s + 23}{s^3 + 5s^2 + 7s + 10} \quad (8)$$

11. Given the transfer function $\frac{10}{s^3 + 3s^2 + 2s}$. Design a feedback controller so that the eigen values of the closed loop system are at $-2, -1 \pm j1$ (16)

12. (a) Explain in detail about the design of state observer for continuous time systems (8)

(b) Develop the state model of a linear system and draw the block diagram of state model. (8)

13. Obtain the eigen values, eigen vectors, modal matrix and Jordan form of the matrix (16)

$$\begin{bmatrix} -4 & 1 & 0 \\ 0 & -3 & 1 \\ 0 & 0 & -2 \end{bmatrix}$$

14. (a) Construct a state model for a system characterized by the differential equation

$$\ddot{y} + 7\dot{y} + 5y + 9y + u = 0 \quad (8)$$

(b) Solve the following homogeneous equation

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \text{ assuming initial condition } X(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad (8)$$

15. Check the observability of the following system.

$$\dot{X} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -2 & -3 \end{bmatrix} X + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u \quad \text{and} \quad Y = [3 \ 4 \ 1]X \quad (16)$$

UNIT – II

z-TRANSFORM AND SAMPLED DATA SYSTEMS

PART – A (2 MARKS)

1. Define Z-transform.

The z-transform of $x(n)$ is denoted by $X(z)$. Its is defined as, $X(z) = \sum x(n)z^{-n}$

Here the summation is from $n = -\infty$ to positive infinity. i.e. both sided

Here z is complex variable. $x(n)$ and $X(z)$ is called z-transform pair.

2. What is meant by Region of convergence?

The region of convergence (ROC) of $X(z)$ is the set of all values of z for which $X(z)$ attains a finite value.

3. What are the properties of ROC?

- i) The ROC is a ring or disk in the z-plane centered at the origin.
- ii) The ROC cannot contain any poles.
- iii) The ROC of an LTI stable system contains the unit circle.
- iv) The ROC must be a connected region.

4. List the properties of z-transform.

- i) Linearity
- ii) Time shifting or translation
- iii) Time reversal.

5. Explain the linear property of z-transform.

If $Z\{x_1(n)\} = X_1(z)$ and

$$Z\{x_2(n)\} = X_2(z)$$

$$Z\{a_1x_1(n) + a_2x_2(n)\} = a_1X_1(z) + a_2X_2(z)$$

6. Explain the time-shifting property of z-transform.

If $Z\{x(n)\} = X(z)$, then

$$Z\{x(n-k)\} = z^{-k} X(z)$$

7. What are the different methods of evaluating inverse z-transform?

- i) Long division method
- ii) Partial fraction method expansion method
- iii) Residue method and iv) Convolution method

8. What is sampled data control system?

When the signal or information at any or some points in a system is in the form of discrete pulses, then the system is called discrete data system or sampled data system.

9. Explain the terms sampling and sampler.

Sampling is a process in which the continuous –time signal (or analog signal) is converted into a discrete time signal by taking samples of the continuous time signal at discrete time instants.

Sampler is a device which performs the process of sampling.

10. What is meant by quantization?

The process of converting a discrete –time continuous valued signal into a discrete –time discrete valued signal is called quantization. In quantization the value of each signal sample is represented by a value selected from a finite set of possible values called quantization levels.

11. State (Shannon's) sampling theorem

Sampling theorem states that a band limited continuous –time signal with highest frequency f_m , hertz can be uniquely recovered from its samples provided that the sampling rate F_s is greater than or equal to $2 f_m$ samples per second.

12. What is zero order hold ?.

The zero order hold is a hold circuit in which the signal is reconstructed such that the value of reconstructed signal for a sampling period is same as the value of last received sample.

13. What is pulse transfer function?

The transfer function of linear discrete time system is called pulse transfer function. It is given by the z-transform of the impulse response of the system. It is also defined as the ratio of Z-transform of output to Z-transform of input of the linear discrete time system. Pulse transfer function = $H(z) = C(z) / R(z)$

14. What are the methods available for the stability analysis of sampled data control systems.?

- (i) Jury's stability test
- (ii) Bilinear transformation
- (iii) Root locus technique.

15. What is bilinear transformation?

The bilinear transformation is a transformation used to map the interior of unit circle in the z-Plane into the left half of s-plane.

16. What are the merits and demerits of sampled data control systems?**Merits**

- i) Systems are highly accurate, fast and flexible
- ii) Use of time sharing concept of digital computer results in economical cost and space.
- iii) Digital transducers used in the system have better resolution.
- iv) The digital components are less affected by noise, non-linearities and transmission errors of noisy channel.

Demerits

- i) Conversion of analog signals to discrete-time signals and reconstruction introduce noise and errors in the signal.
- ii) Additional filters have to be introduced in the system if the components of the system does not have adequate filtering characteristics.

PART – B

1. (a) Solve the following difference equation

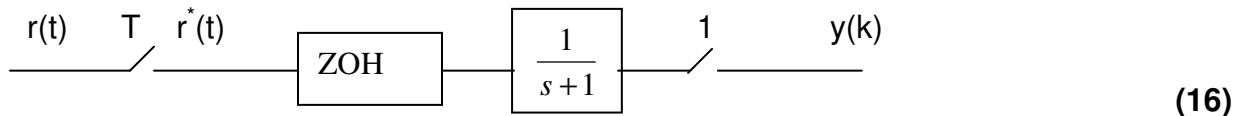
$$2y(k)-2y(k-1)+y(k-2)=r(k)$$

$$Y(k)=0 \text{ for } k < 0 \text{ and}$$

$$r(k)=\begin{cases} 1; k = 0,1,2 \\ 0; k < 0 \end{cases} \quad (8)$$

(b) Check if all the roots of the following characteristic equation lie with in the unit Circle $z^4-1.368z^3+0.4z^2+0.08z+0.002=0$ (8)

2. Derive the transfer function model relating $r(kT)$ and $y(kT)$; $T=0.4$ sec for the following system.



3. (a) Explain the concept of sampling process. (8)

(b) Determine the Inverse z-transform of the following function

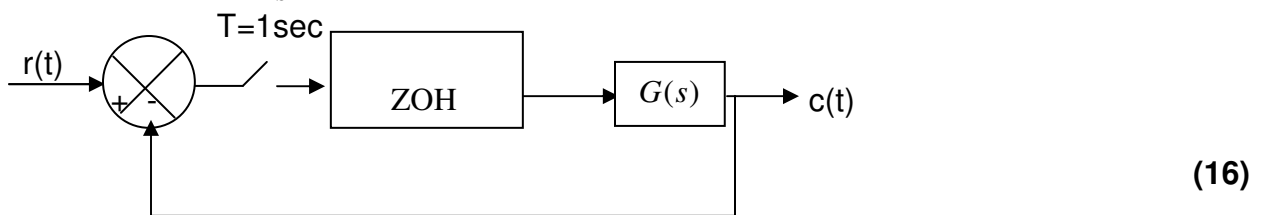
$$(i) F(z) = \frac{1}{1-1.5z^{-1} + 0.5z^{-2}} \quad (ii) F(z) = \frac{1+z^{-1}}{1-z^{-1} + 0.5z^{-2}} \quad (8)$$

4. Solve the difference equation $c(k+2)+3c(k+1)+2c(k) = u(k)$

$$\text{Given that } c(0)=1 ; c(1) = -3 ; c(k) = 0. \text{ for } k < 0 \quad (16)$$

5. For the samples data control system shown in fig. find the response to unit step

$$\text{Input, where } G(s)=\frac{1}{s+1}$$

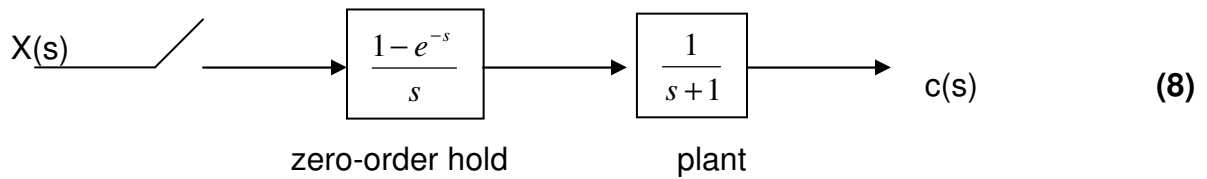


6. Check for stability of the samples data control systems represented by the following characteristic equation

$$(a) 5z^2+-2z+2 \quad (b) z^3-0.2z^2-0.25z+0.05=0 \quad (c) z^4-1.7z^3+1.04z^2-0.268z+0.024=0 \quad (16)$$

7. (a) Obtain the z-transform of $\frac{1}{s(s+1)}$ (4)

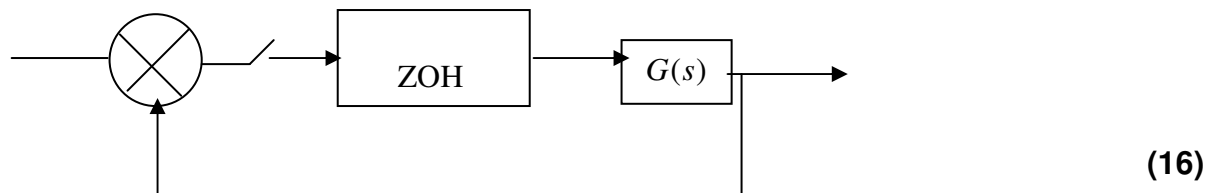
(b) Obtain the pulse transfer function of the system shown in figure



(c) Explain any two theorems on z-transforms. (4)

8. Find the Unit-step response for the first order temperature control system shown

In figure having transfer function of the plant as $G(s) = \frac{4}{s+2}$



9. (a) Determine the Z-domain transfer function for the following s-domain transfer function

$$H(s) = \frac{a}{s^2 - a^2} \quad (8)$$

(b) Determine the inverse z-transform

$$F(z) = \frac{3z^2 + 2z + 1}{z^2 + 3z + 2} \quad (8)$$

10.(a) Explain the signal reconstruction and sampling theorem. (8)

(b) Explain any four z-transform theorems in detail. (8)

11. (a) Explain the impulse response of sampled data system to step and ramp inputs. (8)

(b) Explain Jury's test for a discrete time system and comment your conclusions. (8)

UNIT – III**STATE SPACE ANALYSIS OF DISCRETE TIME SYSTEMS****PART – A (2 MARKS)**

1. Write the properties of the state transition matrix of discrete time systems.

(i) $\Phi(0) = 1$

(ii) $\Phi^{-1}(k) = \Phi^{-1}(-k)$

(iii) $\Phi(k, k_0) = \Phi(k - k_0) = A(k - k_0)$; where $k > k_0$

2. Write the state model of n^{th} order discrete time system.

The state model of a system consists of state equation and output equation. The state model of a n^{th} order discrete time system with m-inputs and p-outputs are

$$X(k+1) = A X(k) + B U(k) \text{ ----- State equation}$$

$$Y(k) = C X(k) + D U(k) \text{ ----- Output equation}$$

Where, $X(k)$ = state vector of order $(n \times 1)$

$Y(k)$ = Output vector of order $(p \times 1)$

$U(k)$ = Input vector of order $(m \times 1)$

A = system matrix of order $(n \times n)$, B = Input matrix of order $(n \times m)$

C = Output matrix of order $(p \times n)$, D = transmission matrix of order $(p \times m)$

3. What are the fundamental elements used to construct the state diagram of discrete time system?

The fundamental elements used to construct the state diagram of discrete time system are scalar, adder, and unit delay unit.

4. A discrete time system is described by the difference equation.

$Y(k+2) + 3y(k+1) + 5y(k) = u(k)$. Determine the transfer function of the system.

Given that, $Y(k+2) + 3y(k+1) + 5y(k) = u(k)$.

On taking Z-transform with zero initial conditions we get,

$$Z^2 Y(z) + 3zY(z) + 5Y(z) = U(z)$$

$$(Z^2+3z+6)Y(Z) =U(Z)$$

$$Y(z) / U(z) = 1 / Z^2+3z+6$$

5. What are the different methods available for computing A^k ?

The various methods available for computing A^k are,

- (i) Computation of A^k using Z-transform.
- (ii) Computation of A^k by canonical transformation.
- (iii) Computation of A^k by Cayley-Hamilton theorem.

6. What is State transition matrix of discrete time system ?

The matrix $\Phi(k)$ is called the State transition matrix of discrete time system.

It is given by $\Phi(k) = z^{-k} \{ (zI - A)^{-1} z \}$. The State transition matrix is used to find the state of the system, at any discrete time instant k .

$$X(k+1) = A X(k) + B U(k)$$

$$Y(k) = C X(k) + D U(k) \text{ .Determine its transfer function.}$$

$$Y(z) / u(Z) = C (zI - A)^{-1} B + D$$

7. What is zero state response?

The output response of a discrete-data system that is due to the input only is called the zero state response; all the initial conditions of the system are set to zero.

8. Define Complete state controllability.

The system described by $X(k+1) = A X(k) + B U(k)$ and $C(k) = D X(k) + E U(k)$ is said to be completely state controllable if for any initial time (stage) $K=0$, there exists a set of unconstrained controls $u(k), k=0, 1, 2, \dots, N-1$, which transfers each initial state $x(0)$ to any final state $x(N)$ for some finite N .

9. Define Complete Observability.

The system described by $X(k+1) = A X(k) + B U(k)$ and $C(k) = D X(k) + E U(k)$ is said to be completely observable if and only if the following $n \times pN$ matrix is of rank n :

Let $L = [D' \ A'D' \ (A')^2D' \ \dots \ (A')^{n-1}D']$ where n is the dimension of $x(k)$ and p is the dimension of $c(k)$. The matrix L is known as the observability matrix.

10. What is the need of observer?

The Observer estimates the state variables from information it receives from the output vector $c(k)$. The output of the observer is the estimated state vector $\hat{x}(k)$.

11. What is pole placement by state feedback?

The pole placement by state feedback is a control system design technique, in which the state variables are used for feedback to achieve the desired closed loop poles.

12. What is the necessary condition to be satisfied for design using state feedback?

The state feedback design requires arbitrary pole placement to achieve the desired performance. The necessary and sufficient condition to be satisfied for arbitrary pole placement is that the system be completely state controllable.

13. What is state observer?

A device (or a computer program) that estimates or observes the state variables is called state observer.

14. What is the necessary condition to be satisfied for design of state observer?

The state observer can be designed only if the system is completely state observable.

15. What is Canonical form of state model?

If the system matrix, A is in the form of diagonal matrix then the state model is called canonical form. In the diagonal matrix, we have the eigen values on the main diagonal and all the other elements are zero.

PART – B

1. (a) Compute state transition matrix Q(k) for the following discrete time system

$$F = \begin{bmatrix} 0 & 1 \\ 0.16 & -1 \end{bmatrix} \quad (8)$$

(b) Obtain the Jordan canonical form realizations for the following transfer function

$$\frac{Y(z)}{R(z)} = \frac{3z^2 - 4z + 6}{\left(z - \frac{1}{3}\right)^3} \quad (8)$$

2. Explain in detail about the linear observer design in discrete time systems. (16)

3. Consider the system $X(k+1) = F x(k) + g u(k)$
 $Y(k) = c x(k)$

$$\text{Where } F = \begin{bmatrix} 0.16 & 2.16 \\ -0.16 & -1.16 \end{bmatrix} ; g = \begin{bmatrix} -1 \\ 1 \end{bmatrix} ; c = [1]$$

Design a state feedback control algorithm which gives closed loop characteristics roots at $0.6 \pm j0.4$ **(16)**

4. A discrete time system has the transfer function $\frac{Y(z)}{U(z)} = \frac{4z^3 - 12z^2 + 13z - 7}{(z-1)^2(z-2)}$

Determine the state model of the system in
 (a) phase variable form (b) Jordan canonical form **(16)**

5. A discrete time system is described by the difference equation

$$y(k+2) + 5y(k+1) + 6y(k) = u(k)$$

$$y(0) = y(1) = 0; T = 1 \text{ sec.}$$

- (a) Determine a state model in canonical form
 (b) Find the state transition matrix
 (c) For input $u(k) = 1; k \geq 1$ find the output $y(k)$. **(16)**

6. Explain the step by step procedure of pole placement by state feedback in discrete systems **(16)**

7.(a) Explain the effect of state feedback on controllability. **(8)**

(b) What is the effect of sampling time on controllability? Explain. **(8)**

8. (a) What is the effect of pole placement by state feedback? **(8)**

(b) Consider the system defined by $X(K+1) = F X(K)$ & $Y(K) = C X(K)$ where

$$F = \begin{bmatrix} -1 & 1 \\ 1 & -2 \end{bmatrix}; \quad C = [1 \quad 0] \quad \text{Design a full order observer. The designed}$$

eigen values for the observer matrix are $\mu_1 = 0.8, \mu_2 = 0.6$. **(8)**

9. Consider the discrete time system described by the state model

$$X(K+1) = F X(K) + gU(K)$$

$$Y(K) = C X(K)$$

Where $F = \begin{bmatrix} 0.9 & 0 \\ 0 & 0.7 \end{bmatrix}; \quad g = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ and $C = [1 \quad 0]$ Design a full order observer. The

designed eigen values for the observer matrix are $\mu_1 = 0.2, \mu_2 = 0.1$. **(16)**

10. Explain in detail about controllability and observability on discrete time systems with examples. **(16)**

UNIT – IV**NON-LINEAR SYSTEMS****PART – A (2 MARKS)****1. What are linear and nonlinear systems? Give examples.**

The linear systems are systems which obeys the principle of super position. The systems which does not satisfy superposition principle are called nonlinear systems.

Example of linear system : $y = ax + b \frac{dx}{dt}$

Example of nonlinear system : $y = ax^2 + e^{bx}$

2. How nonlinearities are introduced in the systems?

The nonlinearities are introduced in the system due to friction, inertia, stiffness, backlash, hysteresis, saturation and dead zone of the components used in the systems.

3. What are the methods available for the analysis of nonlinear system?

The two popular methods of analyzing nonlinear systems are Phase plane method and describing function method.

4. Write any two Properties of nonlinear systems. ?

(i)The nonlinear systems may have jump resonance in the frequency response.

(ii)The output of a nonlinear system will have harmonics and sub harmonics when excited by sinusoidal signals.

5. What is jump resonance?

In the frequency response of nonlinear systems, the amplitude of the response (output) may jump from one point to another for increasing or decreasing values of ω . This phenomenon is called is jump resonance

6. What are limit cycles?

The limit cycles are oscillations of the response (or output) of nonlinear systems with fixed amplitude and frequency. If these oscillations or limit cycles exists when there is no input then they are called zero input limit cycles.

7. What is asynchronous quenching?

In a nonlinear system that exhibits a limit cycle of frequency ω_l , it is possible to quench the limit cycle oscillation by forcing the system at a frequency ω_q , where ω_q and ω_l are not related each other. This phenomenon is called asynchronous quenching or signal stabilization.

8. What is dead-zone?

The dead-zone is the region in which the output is zero for a given input. When the input is increased beyond this dead zone value, the output will be linear.

9. What is describing function?

When the input X to the nonlinearity is a sinusoidal signal ($x=X \sin \omega t$), the describing function of the nonlinearity is defined as

$$\text{Describing function, } K_N(X, \omega) = Y_1 / X (\phi_1)$$

Where, Y_1 = Amplitude of the fundamental harmonic component of the output.

ϕ_1 = Phase shift of the fundamental harmonic component of the output with respect to the input.

X = Maximum value of input signal.

Ω = Angular frequency of input signal.

10. What is autonomous system?

A system which is both free (or unforced or zero input or constant input) and time invariant is called an autonomous system.

11. What is phase plane?

The coordinate plane with the state variables X_1 and X_2 as two axes is called the Phase plane (In phase plane X_1 is represented in x-axis, and X_2 in y-axis.)

12. What is saturation? Give an example.

In saturation nonlinearity the output is proportional to input for limited range of input signals. When the input exceeds this range, the output tends to become nearly constant. Saturation in the output of electronic, rotating and flow amplifiers, speed and torque saturation in electric and hydraulic motors are examples of saturation

13. Write any two properties of non-linear systems.

- i) The non-linear systems may have jump resonance in the frequency response.
- ii) The output of a non-linear system will have harmonics and sub-harmonics when excited by sinusoidal signals.

14. What is Singular point?

A point in phase-plane at which the derivatives of all state variables are zero is called singular point. It is also called equilibrium point.

15. How the singular points are classified?

The singular points are classified as Nodal point, Saddle point, Focus point and Centre or Vortex point depending on the eigen values of the system matrix.

16. Define the stability of a non-linear system at origin.

The autonomous system defined by equation $\dot{X} = F(X)$ is stable at the origin, if for every initial state $X(t_0)$ which is sufficiently close to origin, $X(t)$ remains near the origin for all t .

17. What is the difference in stability analysis of linear and non-linear systems?

In linear system the stability of the system in the entire phase-plane can be judged from the behaviour of the system at equilibrium state (i.e., at singular point), because the linear systems has only one equilibrium state.

In non-linear systems there may be multiple equilibrium states (singular points). The behaviour of non-linear system about the equilibrium point may be different for small deviations and large deviations about the equilibrium point. Hence in non-linear systems, stability is discussed relative to equilibrium state and the general stability of a system cannot be defined.

18. What is phase trajectory?

The locus of the state point (x_1, x_2) in phase plane with time as running parameter is called phase trajectory.

19. What is phase portrait?

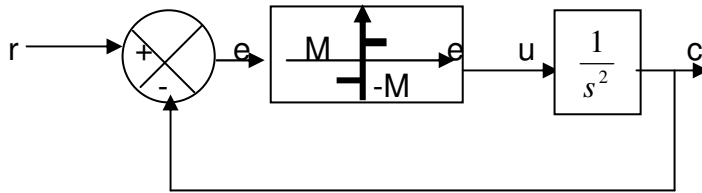
A family of phase trajectories corresponding to various sets of initial conditions is called a phase portrait.

20. What are the methods available for constructing phase trajectories?

- i) Analytical method, ii) Isocline method and iii) Delta method.

PART – B

1. Explain in detail about the behavior of nonlinear system and classifications of Non-linearities. (16)
2. Consider a system with an ideal relay as shown in fig. Determine the singular point. Construct phase trajectories, corresponding to initial conditions. (i) $c(0)=2$; $\dot{c}(0) = 1$ And (ii) $c(0)=2$; $\dot{c}(0) = 1.5$ Take $r=2$ volts & $\mu=1.2$ volts.



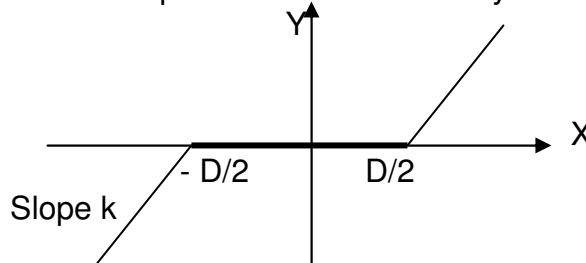
(16)

3. A linear second order servo is described by the equation $\ddot{e} + 2\delta w_n \dot{e} + w_n^2 e = 0$ where $\delta=0.15$, $w_n=1$ rad/sec. $e(0)=1.5$ and $\dot{e}(0) = 0$. Determine the singular point. Construct the phase trajectory, using the method of isoclines. Choose slope as -2.0, - 0.5, 0, 0.5 & 2.0. (16)

4. Derive the describing function for a Backlash non-linearity(16)

5. What is phase plane, phase trajectory and phase portrait?. Draw and explain how to determine the stable and unstable limit cycles using phase portrait? (16)

6. The input-output relationship of dead zone nonlinearity is shown in the figure. The o/p is zero, when the input is less than $D/2$. The input-output relationship is linear when the input is greater than $D/2$. The response of the nonlinearity when input is sinusoid of signal. ($x=X\sin\omega t$)



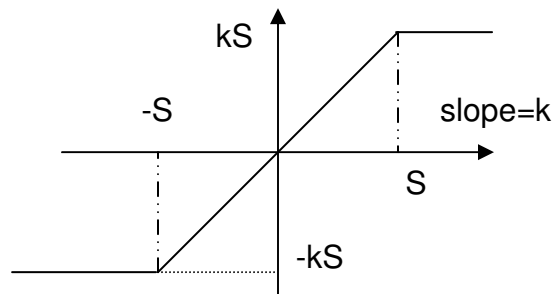
(16)

7. (a) Define liapunov stability. Explain liapunov's direct method (8)

(b) Investigate the stability using liapunov method for a non linear servo system described by the following equation

$$\ddot{E} + k \dot{E} + k_1 (E)^3 + E = 0 \quad (8)$$

8. The input-output relationship of saturation nonlinearity is shown in figure below. The i/p-o/p relation is linear for $x=0$ to s . when the input $x>s$, output reaches a saturated value of ks . The response of the nonlinearity when the input is sinusoidal signal($x=X \sin\omega t$)



(16)

9. A linear second order servo is described by the equation $\ddot{y} + 2\delta\omega_n \dot{y} + \omega_n^2 y = \omega_n^2$ Where

$\omega_n=1$. $y(0)=2.0$ $\dot{y}(0) = 0$. Determine the singular points when (i) $\delta=0$ (ii) $\delta=0.6$ Construct phase trajectory in each case.

(16)

10. Explain the construction of phase trajectory using any two method.

(16)

11. Derive the describing function for relay with deadzone and hysteresis non-linearity

(16)

12. (a) Check the stability of the system described by the state equation using Lyapunov's method.

$$\dot{x}_1 = -x_1 + 2x_1^2 x_2; \dot{x}_2 = -x_2$$

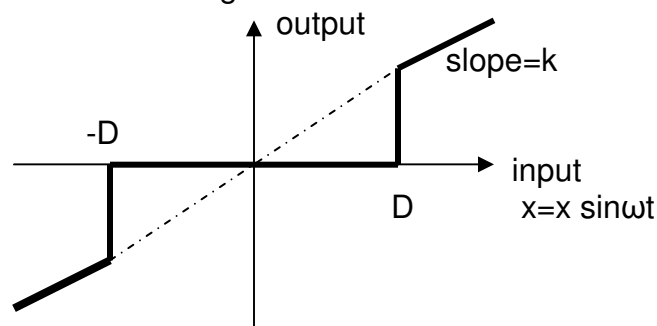
(8)

(b) Check the stability of the Equilibrium state of the system described by the state equation using Lyapunov's method

$$\dot{x}_1 = x_2; \dot{x}_2 = -x_1 - x_1^2 x_2$$

(8)

13. Derive the describing function following function of the element whose input output characteristics are shown in the fig.



(16)

14.(a) State Lyapunov's stability theorem

(8)

(b) Consider the linear autonomous system using lyapunov analysis, determine the stability of the equilibrium state.

(8)

UNIT – V**MIMO SYSTEMS****PART – A (2 MARKS)****1. Define Decoupling.**

It is nothing but Non-interacting. Here every input controls only one output and every output is controlled by only one input.

2. What is Nyquist plot?

This plot is between imaginary and real parts of $G(j\omega)$ or between $\text{Mod } G(j\omega)$ and angle of $G(j\omega)$.

3. What is meant by single valued function?

It is defined as a function in which for each point in the s-plane there is only one corresponding point in the $Q(s)$ plane. here $Q(s) = (s+z_1)(s+z_2) / (s+p_1)(s+p_2)$ which has finite number of zeros and poles.

4. What are singular points?

A function $Q(s)$ is said to be analytic in the s-plane if the function and all its derivatives exist. The set of all points in the s-plane where the function $Q(s)$ or its derivatives $Q'(s)$ does not exist are called singular points. The Poles of the function $Q(s)$ are singular points.

5. What is the effect of on the Nyquist plot if a non-zero pole is added to the transfer function?

Addition of a non-zero pole to the transfer function results in further rotation of the end points of Nyquist plot through an angle of 90° .

6. State the advantages of Nyquist plot?

- i) Closed stability of open loop unstable systems can be obtained.
- ii) Addition of poles to the transfer function reduces the closed loop stability justify by Nyquist plots.

7. What is the effect of zeros of Multivariable systems?

The zeros can have a profound effect upon the shape of the transient response. Thus the placement of closed loop poles does not guarantee the desired transient behaviour.

8.What is MIMO?

Systems with more than one input and/ or more than one output are known as Multi-input Multi-output systems, or they are frequently known by the abbreviation MIMO. This is in contrast to systems that have only a single input and a single output (SISO).

9.What is Model Control?

Optimal control is a particular control technique in which the control signal optimizes a certain “cost index”: for example, in the case of a satellite, the jet thrusts needed to bring it to desired trajectory that consume the least amount of fuel. Two optimal control design methods have been widely used in industrial applications, as it has been shown they can guarantee closed-loop stability. These are Model Predictive control(MPC) and Linear Quadratic-Gaussian control(LQG). The first can more explicitly take into account constraints on the signals in the systems, which is an important feature in many industrial processes. However, the “optimal control” structure in MPC is only a means to achieve such a result, as it does not optimize a true performance index of the closed-loop control system. Together with PID controllers, MPC systems are the most widely used control technique in process control.

10.What are the models of MIMO systems?

MIMO transfer functions are two-dimensional arrays of elementary SISO transfer functions. There are two ways to specify MIMO transfer function models:

- i) Concatenation of SISO transfer function models
- ii) Using transfer function with cell array arguments

PART – B

1.Obtain the minimal realization and Non-minimal realization of the following MIMO system.

$$G_c(s) = \begin{bmatrix} \frac{5}{(s+1)(s+2)} & 0 \\ 0 & \frac{10}{s(s+10)} \end{bmatrix} \quad (16)$$

2. With suitable example, explain the matrix representation of MIMO systems and write its advantages and disadvantages when compared to transfer function representation (16)

3. Explain in detail about model predictive control for MIMO systems. **(16)**

4. Draw the Nyquist plot of the following system and hence obtain gain cross over frequency, phase margin & gain margin.

$$G(s) = \frac{10(s+1)}{s(s+10)(s+20)} \quad (16)$$

5. What is MIMO system? List the various models of MIMO system and explain them in detail. **(16)**

6. Obtain the impulse response of the following MIMO system.

$$G_c(s) = \begin{bmatrix} \frac{10(s+2)}{(s+1)(s+2)} & \frac{-1}{(s+5)(s+10)} \\ \frac{-2}{(s+5)(s+10)} & \frac{s+6}{s(s+15)} \end{bmatrix} \quad (16)$$

7. (a) Distinguish the terms, decoupling zeros, invariant zeros and transmission zeros of a MIMO system. **(8)**

(b) Determine the zeros of the system whose state equation are given by

$$X = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \\ -6 & -11 & -6 \end{bmatrix} X + \begin{bmatrix} 5 \\ 0 \\ 1 \end{bmatrix} u \quad (8)$$

$$Y = \begin{bmatrix} 2 & 3 & 1 \\ 0 & 1 & 1 \end{bmatrix} X$$

8. Draw and explain the basic structure of Model predictive control. **(16)**

9. Explain the following terms:

(a) Decoupling **(8)**

(b) Singular value analysis **(8)**

10. (a) Explain in detail about Multivariable Nyquist plot. **(8)**

(b) Explain the transfer function representation of MIMO Systems. **(8)**
