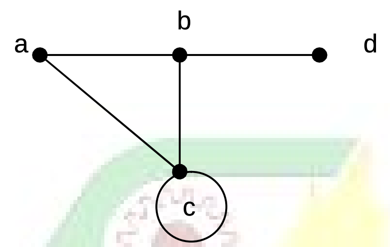
DISCRETE MATHEMATICS

PART-A Questions

1. List A = {1, 2, 3, 4, 5} B = {0, 3, 6}. Find (A – B).
2. Write the converse of the statement. “If I am late, then I did not take the train to work.
3. Write the order of the recurrence relation, $a_{n+2} = a_{n+1}+a_n$, $n \geq 0$, $a_0 = 0$, $a_1 = 1$
4. If p is the permutation, $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 4 & 1 & 2 & 6 & 5 \end{pmatrix}$, find p^{-1}.
5. If L is any Boolean Algebra and x, y ε L, then $(x \wedge y)'$ = ________.
6. The process of visiting each vertex of a tree in some specified order is called ________.
7. For the given graph, find the degree of the vertex ‘C’.



8. Define an Euler path in a graph.
9. Find the distance between the code words: x = 1001, y = 0100
10. Define finite state machine.
11. $A \text{ and } B$ are two sets. Find $(A-B)\cap(B-A)$
12. Give the truth table for the bi-conditional statement $P \leftrightarrow Q$
13. If the set A has n elements ,how many relations are there from A to A
14. $f: R \rightarrow R$ is given by $f(x) = x^2$. Is f is a one to one map? Where R is the set of real numbers.
15. Say whether the relation > on the set of positive integers is a partial order relation
16. True or False “The spanning tree of a graph is unique”.
17. Define Euler graph
18. Give an example of a semi group
19. Define group code
20. Draw the digraph of the machine whose state transition table is given below.

	0	1
S_0	S_0	S_1
S_1	S_0	S_1

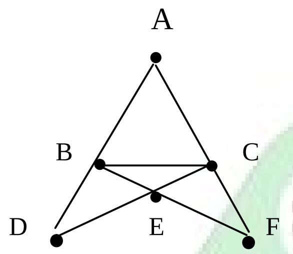
21. Give an example of infinite sequence
22. Let A be a set with n elements .How many subjects does A have?
23. Let A=Z, the set of integers, and let $R = \{(a,b) \in A \times A / a \leq b\}$. Is R an equivalence relation?

24. When a permutation of finite set is called even permutation?
25. Determine whether the relation R is a partial order on the set
    $A = Z$ (*the set of* integers), *and* aRb if and only if $a = 2b$
26. Give the tree representation for the expression $(a+b)c-d$
27. Define Hamiltonian graph
28. Give an example of a semi group which has no identity element
29. Find the weight of the code word $x = 110101$
30. Define equivalent Finite State Machines
31. What can you say about A and B if $A = \{1,2,3\}$ and $B = \{x / x > 0 \text{ and } x^2 < 12\}$.
32. Express the specifications "the automated reply cannot be sent when the file system is full" using logical connectives.
33. Define an equivalence relation.
34. Let f be a function from Z to Z with $f(x) = x^2$. Is f invertible?
35. Define lattice.
36. Define weighted graph.
37. Define an Euler circuit.
38. Give an example of not a semi group.
39. Find the weight of x=10100 in B^5.
40. Define finite state machine.
41. Give an example of empty set
42. True or False $p \vee q \Rightarrow p$
43. If the set *A* has *m* elements and the set *B* has *n* , then how many relations are there from A to *B*
44. $f : R \to R$ is given by $f(x) = x^2$. Is f is a one to one map? Where R is the set of real numbers.
45. Say whether the relation < on the set of positive integers is a partial order relation.
46. True or False "The sum of the degrees of all vertices of a graph is equal to the total number of edges ".
47. Define Hamiltonian path
48. The set Z(set of all integers) with the binary operation subtraction is a semi group or not
49. Define group code
50. Draw the digraph of the machine whose state transition table is given below.

	a	b
S_0	S_1	S_0
S_1	S_0	S_1

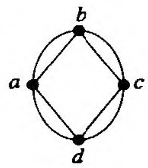
51. Give the truth table for p $\to$ q.
52. If A has five elements. Find the cardinality of the power set of A.
53. Let A=Z, the set of integers, and let $R = \{(a,b) \in A \times A / a \le b\}$. Is R an equivalence relation?
54. Suppose R and S are relations on a set A. Prove that if R and S are reflexive, then $R \cap S$ is reflexive.

55. Define: Partial order relation.
56. Draw a binary tree with five vertices.
57. Give an example of Euler graph.
58. Define: Semi-group.
59. Define: Type-2 grammar.
60. Define group code.
61. If the cardinality of a set A is n, what is the cardinality of the power set p(A)?
62. Give the truth table for $p \Leftrightarrow q$.
63. If p is the permutation $p = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 4 & 1 & 2 & 6 & 5 \end{pmatrix}$, find p^{-1}.
64. Define an equivalence relation.
65. If L is any Boolean Algebra and $x, y \in L$ then $(x \vee y)'$ = __________
66. A tree with n vertices has _______edges.
67. Define a Hamiltonian path in a graph.
68. Find an Euler circuit in the given graph.

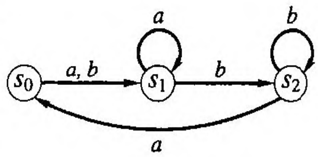


69. Find the distance between the code words x = 1101, y = 0110.
70. Define a moore machine
71. The finite sequence defined by $c_1 = 5$, $c_n = 2\, c_{n-1}$, $2 \le n \le 6$ is __________.
72. State the Pigeon Hole Principle.
73. Give any one partition of the set {a, b, c, d, e}.
74. Let A be a set of people. Is the relation R = {(a,b) $\in$ A x A / a is a cousin of y}, transitive?
75. State the idempotent properties of a lattice.
76. Give the logic diagram of p(x,y,z) =$(x \wedge y) \vee z'$.
77. Define degree of a vertex in a graph.
78. Give an example of a monoid.
79. Define Group code.
80. Define state transition function.
81. What is the binary representation of 53?
82. Write in symbolics : When you sing my head hurts.
83. If R = { (1,2), (1,3), (2,4), (3,3)} is a relation from A = {1, 2, 3}to B = = { 2, 3, 4} find the complement of R.
84. If the matrix of the relation R on a set A = {a, b, c} is given by $\begin{pmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \end{pmatrix}$ list the elements of R.
85. What is the least element of the poset of nonnegative real numbers with partial order '≤' ?
86. How many edges does a tree with10 vertices have?

87. What is meant by a monoid?
88. What is meant by a Hamiltonian circuit?
89. Find the distance between x = 11010010 and y = 00100111.
90. What is meant by context free grammar?
91. Choose four cards at random from a standard 52 card deck. What is the probability that four kings will be chosen?
92. Make a truth table for $(p \wedge q) \vee (\sim p)$
93. Let A = {1,2,3} and B = {a,b,c}. Let R be a relation defined from A to B as R = {(1,a), (1,b), (2,b), (2,c)}. Find R^{-1}.
94. If f: $Z \rightarrow Z$ and g: $Z \rightarrow Z$ are defined as f(a) = a+1 and g(b) = 2b, find $g \circ f$.
95. Is $(Z^+, <)$ a poset?
96. Give the tree representation of the expression a + (b – c) d.
97. Define: Regular graph.
98. Define chromatic number of G.
99. Find the distance between x = 110110 and y = 000101
100. Define Type 2 pharse structure grammar.
101. Write the formula with out conditional statement $(P \Leftrightarrow Q) \Rightarrow R$.
102. State: Pigeonhole Principle.
103. Let R be the relation on A = {2, 3, 4, 9} defined by "x is relatively prime to y". Write R as the set of order pairs (x, y).
104. Determine the Θ – class of the polynomial function f (n) = $4n^4 - 6n^7 + 25n^3$.
105. Draw the Hasse diagram for divisibility on the set {1, 2, 3, 5, 11, &13}.
106. Determine the Root if R is a Tree R = {(a, d), (b, c), (c, a), (d, e)}.
107. Does the graph has Euler Graph? If so, give one Example:



108. The Identity Element of the Semi group $(Z^+, +)$ is ____________
109. Construct the State transition Table of the Finite State machine Whose Digraph is as shown below:



110. Find the Weight of the following words in B^5 (i) 00100 (ii) 10001.
111. If A and B are two sets, define their symmetric difference.
112. Determine the truth or falsity of the statement "2<3 or 3 is not a positive integer".
113. Given A = {a, b,c} and R = {(a,a), (a,b)}, is R transitive?
114. If f(x) = 2x and g(x) = x^3+2x-3, find $f \circ g(x)$
115. Give an example of a Boolean algebra having nine elements.
116. Name one algorithm used to find minimal spanning tree.
117. What is the chromatic number of the complete graph with four vertices?
118. What is the difference between a monoid and a semi-group?

119. Which notation is commonly used to display productions for type 2 grammars?
120. Find the weights of x = 01000 and y = 11100
121. The finite sequence defined by $c_1 = 5$, $c_n = 2\, c_{n-1}$, $2 \le n \le 6$ is ________.
122. State the Pigeon Hole Principle.
123. Give any one partition of the set {a, b, c, d, e}.
124. Let A be a set of people. Is the relation R = {(a,b) ∈ A x A / a is a cousin of y}, transitive?
125. State the idempotent properties of a lattice.
126. Give the logic diagram of $p(x,y,z) = (x \wedge y) \vee z'$.
127. Define degree of a vertex in a graph.
128. Give an example of a monoid.
129. Define Group code.
130. Define state transition function.
131. Let u = {1, 2, 3, 4, 5, 6} and A = { 1, 2}. Find $f_A(3)$,where f_A is the characteristic function.
132. Describe the infinite sequence –4, 16, –64, 256 ,……….. using explicit formula.
133. Let A = {a, b, c, d, e, f, g, h} consider the following subsets of A: A_1= {a, b, c, d}, A_2 = {a, c, e,f, g, h}, A_3 = {a, c, e, g}, A_4 = {b,d} A_5 = {f, h}. Give a partition P of A.
134. Determine whether a relation R on A = Z, aRb if and only if a+b is even is reflexive.
135. Define partial order on a set A.
136. A tree with n vertices has ________ edges.
137. Write the chromatic number of the complete graph K_n.
138. On z^+, $a*b = a^b$ is a binary operation. True / False.
139. Draw the digraph of the machine whose state transition table is given below:

	0	1
s_0	s_0	s_1
s_1	s_1	s_2
s_2	s_2	s_0

140. Let d be the (4, 5) decoding function. Determine d (10010)
141. If A is a subset of the null set ϕ, Prove that A = ϕ.
142. Construct the truth table of the formula $(\neg p \vee q) \wedge (\neg q \vee p)$
143. Let A = {a, b, c, d} and R the relation on A that has the matrix

$$M_R \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

Construct the digraph of R.

144. Show that the function $f : R \to R$ given by $f(x) = \sin x$ is neither 1 – 1 nor onto.
145. Draw the Hasse diagram of (X, ≤), where X is the set of positive divisions of 45

and the relation $\leq$ is such that $\leq = \{(x, y); x \in A, y \in A \wedge (x \, divides \, y)\}$

146. Define an equivalence relation with an example.
147. In the group {2, 4, 6, 8} under multiplication module 10, what is the identity element?
148. For any group G, if $a^2 = e$ with $a \neq e$, then prove that G is Abelian.
149. Define a Phrase structured Grammar.
150. If $x = <1,0,0,1>, y = <0, 1, 0, 0>$, Find H(x,y)?
151. A = {1, 2, 3}. Number of proper subsets of A is _________.
152. The number of distinguishable 'Words' that can be formed from the letters of MISSISSIPPI is _________.
153. If A = (1, 2, 3) and B = {r, s}, find A x B.
154. $M_R = \begin{array}{c} \\ 1 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{matrix} 1 & 4 & 5 \end{matrix} \\ \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \end{array}$ Draw the digraph.
155. Define partial order relation.
156. Let L be a lattice, then for every a, b in L avb = b if and only if a ≤ b. (True / False)
157. A vertex with degree _________ will be called as an isolated vertex.
158. Define semi group.
159. Find the distance between x and y, x = 110110 and y = 000101.
160. Find the weight of each of the following words in B^5 i) x = 01000 ii) y = 11111
161. Write the dual of $(P \vee Q) \wedge R$.
162. Define contradiction.
163. Let X = {1, 2, 3}, Y = {3, 4}. Find $X \times Y$.
164. Let X = {1, 2, 3, 4} and R = {(1, 1), (1, 4), (4, 1), (4, 4), (2, 2), (2, 3), (3, 2), (3, 3)}. Write the matrix of R and sketch its graph.
165. Define modular lattice.
166. Define Boolean algebra.
167. Define a connected graph.
168. Define isomorphic graph.
169. Define equivalent Finite State Machines
170. Find the distance between x and y if x = 110110 and y = 000101.
171. State De Morgan's laws of set theory.
172. State Pigeon-hole principle.
173. Define a binary relation from one set to another.
174. If f:R $\rightarrow$R is given by f(x) = 3x – 7 , find f^{-1}.
175. State the isotonic property of a lattice.
176. What is meant by minimum spanning tree?
177. Give an example of a graph, which contains an Eulerian circuit that is also a Hamiltonian circuit.
178. Find the identity element of the algebraic system {S,*} where S is the set of integers and * is defined by a*b = a+b+2 , for all a, b$\in$S.
179. How will you identify the accepting states of the finite state automaton from

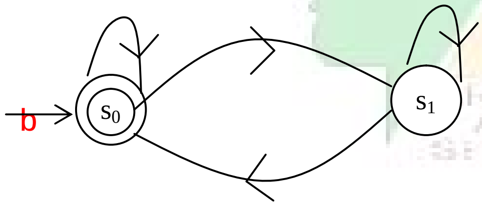
the state diagram of the corresponding finite state machine?

180. What is group code?
191. Define the join of two m x n Boolean matrices.
192. Give the contrapositive of the implication “If it is raining, then I get wet”.
193. Let A={1,2,3,4} and let R={(1,2),(2,2),(3,4),(4,1)}. Is R antisymmetric? Yes / No.
194. Let A={1,2,3} and p=$\begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix}$ be one of the 6 possible permutations. Find p^{-1}.
195. Define Boolean algebra.
196. Define binary tree.
197. Is the set Z with the binary operation of subtraction a semigroup? Yes / No.
198. Define Euler path.
199. Define group code.
200. Define Finite state machine.

PART-B Questions

1. An urn contains 15 balls, eight of which are red and seven are black. In how many ways can five balls be chosen so that two are red and three are black ?
2. Determine whether the relation *R* on the set *A* is an equivalence relation where $A=\{a,b,c,d\}, R=\{(a,a),(b,a),(b,b),(c,c),(d,d),(d,c)\}$
3. Is the poset $A=\{2,3,12,24,36,72\}$ under the relation of divisibility a lattice?
4. Give an example of a graph that needs 4 colors
5. Is the string $\alpha = abab$ is accepted by the Finite State Machine given below

b a



a

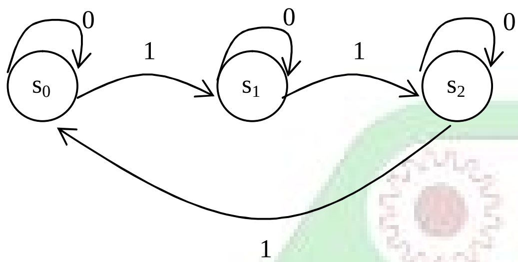
6. A box contains 7 red balls and 5 green balls. If 5 balls are selected from the box at random ,what is the probability that two of the selected balls will be red and three of the selected balls will be green
7. $A=\{1,2,3,4\}, R=\{(1,1),(1,2),(2,1),(2,2),(2,3),(2,4),(3,4),(4,1)\}$.Draw the digraph of *R*.
8. Let $A=\{1,2,3,4,12\}, define$ the partial order $a \le b$ if and if only $a \ divides\ b$.Draw the Hasse diagram of the poset $(A,\le)$
9. Give an example of a graph that needs 4 colors
10. Define finite state machines
11. Compute the truth table for the statement. $p \to q \Leftrightarrow \quad q \to \quad p$.
12. Give the relation R on A and its digraph, where A = {1, 2, 3, 4} and

$$M_R = \begin{pmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 \end{pmatrix}.$$

13. Let A = {u, v, w, x, y , z} and R = {(u, x), (u, v) , (w, v), (x, z), (x, y)} Check whether R is a tree, if it is, find the root.
14. Define chromatic number of a graph G. Write the chromatic number and the chromatic polynomial of the complete graph k_4.
15. Draw the digraph for the given transition table.

	0	1
S_0	S_0	S_1
S_1	S_1	S_2
S_2	S_2	S_0

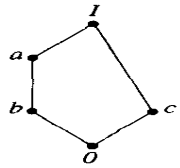
16. Compute the truth table for $(p \Rightarrow q) \Leftrightarrow (q \Rightarrow p)$
17. Let A = {a, b} R = {(a, a) , (b, a) , (b, b) } S = {(a, b) , (b, a), (b, b)} find $R \cup S$.
18. Define (a) height (b) leaves of a tree.
19. Find the chromatic polynomial and chromatic number of k_3.
20. Give the transition table for the given digraph.



21. A die is tossed and the number showing on the top face is recorded. Let E: The number is at least 3; F: The number is at most 3; G: The number is divisible by 3
    (a) Are E and F Mutually Exclusive?
    (b) Are F and G Mutually Exclusive?
22. Let A = {1,2,3}. Let $P_1 = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix}$ and $P_2 = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}$ be two permutation functions. Find $P_2 \circ P_1$ and P_2^{-1}.
23. Give the labeled binary tree representation of (3 – 2x) + ((x – 2) – (3+x)).
24. Draw K_5 and give its matrix representation.
25. What are the three types of pharse structure grammar?
26. Make the truth table for the statement $(p \wedge q) \vee \sim r$
27. If A = {1,2,3,4} and R = {(1,1), (1,2), (2,1), (2,2), (2,3), (2,4), (3,4), (4,1)}, represent R by a matrix.
28. Define dictionary/lexicographic order on the set of English alphabets and illustrate with an example.
29. Let G be a group, and let a and b be elements of G. Then show that the equation ax=b has a unique solution in G.
30. Define Hamming distance δ(x,y) between two words in B^m and find the Hamming distance between x = 110110 and y = 000101.
31. Test the formula for tautology or contradiction $Q \vee (P \wedge \neg Q) \vee (\neg P \wedge \neg Q)$
32. Let A = {1, 2, 3, 4}, and R be the relation on A that has the Matrix M_R. Construct the Digraph of R, and List in-degrees and out-degrees of all vertices.

$$M_R = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$

33. Verify whether the lattice is Distributive or not.

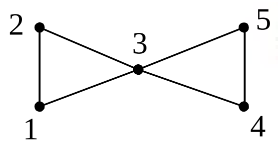


34. Prove: If a Graph G has more than two vertices of odd degree, then there can be no Euler path in G.
35. Let G be the Grammar with V = {a, b, c, S}, T = {a, b, c}, starting symbol S, and Production S → abS / bcS / bbS / a / cb. Construct the Parse Tree bbbcbba.
36. Prove that $\sqrt{2}$ is irrational by a suitable method of proof.
37. Let A_1 = set of all even integers and A_2 = set of all odd integers. Show that $\{A_1, A_2\}$ is a `partition of Z.
38. Simplify $(y \wedge z) \vee x' \vee (w \wedge w') \vee (y \wedge z')$ to an expression with two variables.
39. Define the following properties of a relation: antisymmetric, asymmetric.
40. Briefly explain BNF notation.
41. Show that if 30 dictionaries in a library contain a total of 61,327 pages, then one of the dictionaries must have at least 2045 pages.
42. If Z is the set of integers does $\{A_1, A_2\}$ form a partition of Z where A_1 denotes the set of positive integers and A_2 the set of negative integers? Justify.
43. Define a spanning tree.
44. If a, b ϵ G where G is a group, show that $(ab)^{-1} = b^{-1}a^{-1}$.
45. Draw the digraph of the machine whose state transition table is given below

	0	1
S_0	S_0	S_1
S_1	S_1	S_2
S_2	S_2	S_0

46. Symbolize the expression "All the world loves a lover".
47. If R is the relation on the set of integers such that $(a,b) \in R$, if and only if $3a + 4b = 7n$ for some integer *n*, Prove that R is an equivalence relation.
48. Let a relation R be defined on the set of all real numbers by 'if *x, y* are real numbers, $xRy \Leftrightarrow x-y$ is a rational number', show that R is an equivalence relation.
49. Prove that the necessary condition that a non – empty subset H of a group G be a subgroup is $a \in H$, $b \in H \Rightarrow a*b^{-1} \in H$.
50. Construct a Phrase – structure grammar that generates the set $\{0^n 1^n / n \geq 0\}$.
51. Compute the truth table of the statement $(p \Rightarrow q) \Leftrightarrow (\sim q \Rightarrow \sim p)$.
52. Let R and S be relations on A if R and S are transitive then $R \cap S$ is also transitive. Prove it.
53. Let L be a lattice, then prove av (bvc) = (avb)vc.

54. Let G be a group. Prove each element in G has one inverse in G.
55. Define finite state machine.
56. Show that $(P \to Q) \rightleftarrows (\neg P \vee Q)$.
57. Let A = {1, 2, 3, 6}. Examine whether 'divides' is a partial ordering relation on A.
58. Prove $(a + b)^* = a^*(ba^*)^*$.
59. Show that in a simple digraph, the length of any elementary path is less than or equal to n–1, where n is the number of nodes in the graph.
60. Use the grammar G given as: G = {{S, A, B}, {a, b}, P. S},
P = {(S → AB), (S → bA),(A → a), (A → as), (A → bAA), (B → b), (B → bS), (B → aBB)} to construct the derivation tree for the string aaabbb.
61. Compute A∨B, A∧B, A⊙B given $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$; $B = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$
62. Let A = { 1, 2, 3, 4} = B; aRb if and only if a ≤ b
    (i) Write R
    (ii) Give matrix of R
    (iii) Draw digraph of R
63. Construct the tree of algebraic expression $(7+(6-2)) - (x-(y-4))$
64. Define (i) Euler path (ii) Hamiltonian path
(iii) Give Euler circuit of the graph:



65. Consider the finite state machine whose state transition table is given below.

	0	1
S_0	S_0	S_1
S_1	S_2	S_2
S_2	S_1	S_0

List the values of the transition function f_w where w =011.
66. Show that the statement (p v q)⇔ (q v p) is a tautology using truth table method.
67. Let A={1,2,3,4} and let R={(2,1),(2,3),(3,2),(3,3),(2,2),(4,2)}. Find the reflexive closure of R and Symmetric closure of R.
68. Write the algorithm for Inorder in Tree search.
69. Define Hamiltonian circuit and find the Hamiltonian circuit in K_4.
70. Show that the string abaab accepted by the Moore machine whose state transition table is given below:

	a	b
S_0	S_0	S_1
S_1	S_1	S_2
S_2	S_2	S_2

where S_0 is start state and S_2 is final state.
71. Construct a truth table for $(\neg P \leftrightarrow \neg q) \leftrightarrow (p \leftrightarrow q)$.

72. Draw the digraph representing the partial ordering{ (a, b)/ a divides b } on the set {1,2,3,4,5,6,7,8}.
73. In a Boolean Algebra, if a + b = 1 and a. b = 0, show that b = a′.
74. If all the vertices of an undirected graph are each of odd degree k, show that the number of edges of the graph is a multiple of k.
75. Draw the state diagram of a Finite state Automaton that accepts strings over {a, b} containing exactly one b.

PART-C Questions

1. a. Show that $R \wedge (P \vee Q)$ is a valid conclusion from the premises

$$P \vee Q, Q \to R, P \to M \text{ and } \sim M$$

   b. Solve the recurrence relation: $S(k) + 3S(k-1) - 4S(k-2) = 0, S(0) = 3, S(1) = -2$
2. a. By mathematical induction prove that $n! \geq 2^{n-1}$,for all $n \geq 1$

   b. Using Euclidean algorithm find the greatest common divisor and the least common multiple of the integers 72 and108 .
3. a. Let $A = \{1,2,3,4\}$ and R be the relation on A that has the matrix

$$M_R = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

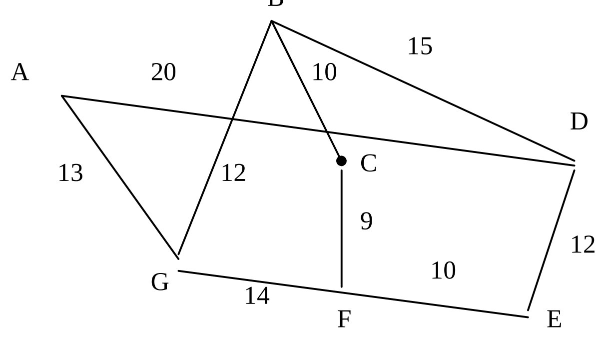
   Construct the digraph of R and list in-degree and out- degree of all vertices

   b. Show that the function $f : \{R-(3)\} \to \{R-(1)\}$ given by $f(x) = \dfrac{x-2}{x-3}$ is a bijection and find its inverse

4 a. Using Warshall algorithm, find the transitive closure of the relation $R = \{(1,2),(2,3),(3,3)\}$ On the set $A = \{1,2,3\}$

   b. Let $X = \{1,2,3,4,5,6,7\}$ and $R = \{(x,y) / (x-y) \text{ is divisible by } 3\}$.Show that R is an equivalence class, find them also draw the graph of R
5. a. Let $p(x,y,z) = ((x \wedge y) \vee (y \wedge z'))$. Give the truth table for the corresponding function $f : B_3 \to B$ and the logic diagram for p

   b. Find the minimal spanning tree for the following graph



6. a. Use Karnaugh map simplify the Boolean expression

$$f(a,b,c,d) = a'b'c'd' + ab'c'd' + a'bc'd' + a'b'cd + a'bcd + abcd + ab'cd + a'b'cd' + a'bcd' + abcd' + ab'cd'$$

   b. Give the tree representation of the expression $(3-(2\times x))+((x-2)-(3+x))$

7. a. Prove that a connected graph G is an Euler graph iff all vertices of G are of even degree .

   b. Let G be the set of all nonzero real numbers and let $a*b=\frac{ab}{2}$.Show that $(G,*)$is an abelian group

8. a. Give an example of a graph which is Hamiltonian but not Eulerian and vice-versa.

   b. $H=\{[0],[3]\}$ is a sub group of $(Z_6,+_6)$. Find the quotient group Z_6/H

9. a. Use mathematical induction to show that $\frac{1}{1.2}+\frac{1}{2.3}+\frac{1}{3.4}+\dots\dots\dots\dots+\frac{1}{n(n+1)}=\frac{n}{n+1}$.

b. In a survey of 260 college students, the following data were obtained. 64 had taken mathematics course, 94 had taken computer science course and 58 had taken business course. 28 had taken both mathematics and business courses, 26 had taken mathematics and computer science courses, 22 had taken both computer science and business courses and 14 had taken all the three courses.
   i. How many students had taken none of the three courses?
   ii. How many students had taken only computer science?

10. a. Prove that $((p\to q)\wedge(q\to r))\to(p\to r)$ is a tautology.

   b. Solve the recurrence relation, $u_n = 2\,u_{n-1} - u_{n-2}$ with initial conditions, $u_1 = 1.5$ and $u_2 = 3$.

11. a. Let S = {1, 2, 3, 4} and A = S x S. Define the relation R on A by (a, b) R (a′, b′) if and only if a + b = a′ + b′.
   i. Show that R is an equivalence relation.
   ii. Compute A/R

   b. Let A = {1, 2, 3, 4, 5, 6} and P = $\begin{pmatrix}1&2&3&4&5&6\\2&4&3&1&5&6\end{pmatrix}$ be a permutation of A.
   i. Compute p^2.
   ii. Write p as a product of disjoint cycles.
   iii. Determine whether p is even or odd.

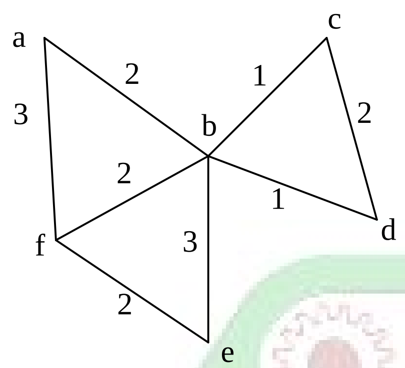
12. a. Let A = $\{a_1, a_2, a_3, a_4\}$ and let R be a relation on A, whose matrix is

$M_R = \begin{pmatrix}0&1&0&0\\1&0&1&0\\0&0&0&1\\0&0&0&0\end{pmatrix}$. Find the transitive closure of R using Warshall's algorithm.

   b. Let A = B = R, the set of real numbers. Let f : A → B is defined by $f(x) = 2x^3 - 1$

and g : B → A is defined by $\sqrt[3]{\frac{y+1}{2}}$. Show that f is a bijection between A and B.

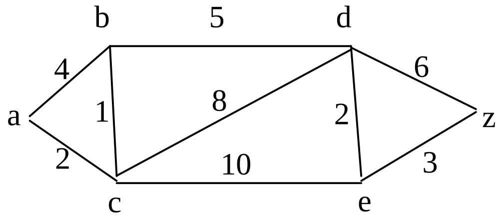
13. a. Consider the Boolean polynomial p (x, y, z) = (x ∧ y′) ∨ (y ∧ (x′ v y)). If B = [0, 1], compute the truth table of the function f : B_3 → B defined by p.
    b. Construct the Karnaugh map for the function f and find the Boolean expression for f. s (f) = {(0, 0, 1) , (0, 1, 1) , (1, 0, 1) , (1, 1, 1)}
14. a. Find the ordered rooted tree representing the expression ((x+y)↑2)+((x-4)/3). Hence obtain the prefix and postfix forms of the expression.
    b. Define spanning tree. Explain Kruskal's algorithm for producing a minimum spanning tree and use it to obtain a minimum spanning tree for the following graph.



15. a. Show that $((p \vee q) \wedge (\neg p \vee r)) \rightarrow (q \vee r)$ is a tautology.
    b. Show that the premises "A student in this class has not read the book", and "Everyone in this class passed the first exam" imply the conclusion "Someone who passed the first exam has not read the book".
16. a. Determine the number of integers between 1 and 500 that are divisible by 2, 3 or 5.
    b. Use Karnaugh map to simplify the expression
    $wx\overline{y}\,\overline{z} + w\overline{x}yz + w\overline{x}y\overline{z} + w\overline{x}\,\overline{y}\,\overline{z} + \overline{w}x\overline{y}\,\overline{z} + \overline{w}\,\overline{x}y\overline{z} + \overline{w}\,\overline{x}\,\overline{y}\,\overline{z}.$
17. a. Prove that $1^2 + 3^2 + 5^2 + \ldots\ldots + (2n+1)^2 = \frac{(n+1)(2n+1)(2n+3)}{3}$ by mathematical induction.
    b. Give the recursive definition of Fibonacci numbers. Also construct a recursive algorithm for Fibonacci numbers. Hence find the Fibonacci numbers f_2, f_3, f_4, f_5 and f_6.
18. a. If R = {(1,2),(1,3),(2,3),(2,4),(3,1)} and S = {(2,1),(3,1),(3,2),(4,2)} are relations on A = {1,2,3,4}. Find R∪S, R∩S, SoR, R^{-1} and $\overline{S}$.
    b. Find the transitive closure, symmetric closure of the relation R = {(1,1),(1,2),(2,1),(3,2)} on the set A = {1,2,3}.

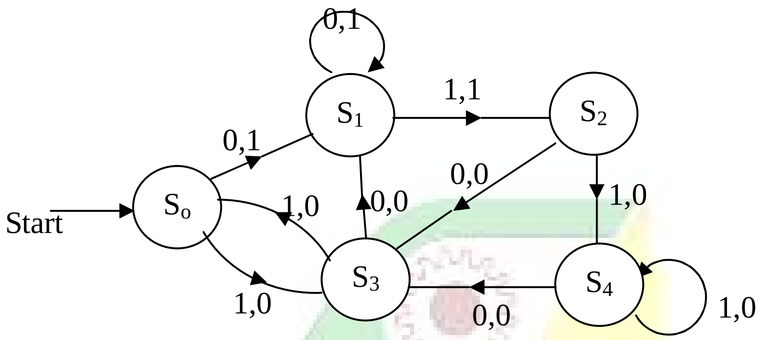
19. a. Define connected undirected graph. Prove that there is a simple path between every pair of distinct vertices of a connected undirected graph.
    b. Prove that a connected multigraph has an Euler path if and only if it has exactly two vertices of odd degree.
20. a. Use Dijkstra's algorithm to find a shortest path between a and z in the

following graph.

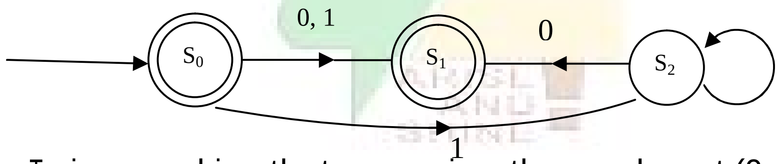


b. Define planar graph. Show that the graph $K_{3,3}$ is not planar.

21. a. Explain the four types of grammars.
    b. Give the formal definition of a finite state machine with output. Construct the state table for the FSM with state diagram shown below. Find also the output string for the input string 101011.



22. a. Find a DFA that recognizes the same language as the NDFA, whose state diagram is given below.



    b. Find a Turing machine that recognizes the regular set $(0\cup1)1(0\cup1)^*$.

23. a. Show by mathematical induction that for all $n \geq 1,\ 1+2+3+....+n = \frac{n(n+1)}{2}$.
    b. A survey of 500 television watchers produced the following information: 285 watch football games, 195 watch hockey games, 115 watch basket ball games, 70 watch foot ball and hockey games, 50 watch hockey and basket ball, 45 watch foot ball and basket ball and 50 do not watch any of the 3 games. (i) How many people in the survey watch all the three games. (ii) How many people watch exactly one of the three games.

24. a. Show that $Q \vee (P \wedge \neg Q) \vee (\neg P \wedge \neg Q)$ is a tautology.
    b. Define recurrence relation and solve the recurrence relation $d_n = 2d_{n-1} - d_{n-2}$ with initial condition $d_1 = 1.5$ and $d_2 = 3$.

25. a. Let a relation R be defined on the set of all real numbers by if x, y are real numbers $xRy \Leftrightarrow x - y$ is a rational number. Show that R is an equivalence relation.
    b. Let $f: R \rightarrow R$ and $g: R \rightarrow R$ where R is the set of real numbers be given by

$f(x) = x^2 - 2$ and $g(x) = x + 4$. Find $f \circ g$, $g \circ f$. State whether these functions are injective, surjective and bijective.

26. a. Let A = {1,2,3,4} and R={(1,1),(1,3),(2,3),(3,4),(4,1),(4,2)}. Find the transitive closure of R.
    b. Let A = {1,2,3,4} and R={(1,2),(2,3),(3,4),(2 1)}.Using Warshall's algorithm find the transitive closure of R.

27. a. Let S = {a,b,c}and A=P(S). Draw the hasse diagram of the poset A with the partial order$\subseteq$.
    b. State and prove DeMorgan's law in Boolean algebra.

28. a. Let L be a distributive lattice and $a,b,c \in L$. If $a \wedge b = a \wedge c$ and $a \vee b = a \vee c$ then b=c.
    b. Describe the addition of two one digit binary numbers.

29. Prove that a given connected graph G is Eulerian iff all vertices of G are of even degree.

30. Let G be a group with identity element e, let a ϵ G of order n. Let m be an integer then prove that a^m =e iff n divides m.

31. a. Construct a finite automaton accepting all strings of 0's and 1's having both an odd number of 0's and an odd number of 1's.
    b. Construct a grammar for the language $L(G) = \{a^n b a^m / n, m \geq 1\}$.

32. a. Show that (m, m+1) parity check code $e : B^m \to B^{m+1}$ is a group code.
    b. For $H = \begin{bmatrix} 1\,1\,0 \\ 1\,0\,1 \\ 0\,1\,1 \\ 1\,0\,0 \\ 0\,1\,0 \\ 0\,0\,1 \end{bmatrix}$. Find $e_H : B^3 \to B^6$, Form the decoding table.

33. a. By mathematical induction prove that the sum of first n positive odd integers is n^2
    b. Show that $S \wedge J$ is a valid conclusion from the premises
    $$P \to Q, Q \to \neg R, R, P \vee (J \wedge S)$$

34. a. Let $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$ Compute $A \vee B$ and $A \wedge B$
    b. Solve the recurrence relation: $S(k) - S(k-1) - 2S(k-2) = 0, S(0) = 0, S(1) = 1$

35. a. Let $A=\{1,2,3,\}$,and $R=\{(1,1),(1,3),(2,2),(3,1),(3,2)\}$. Find the transitive closure of *R*

b. Show that the permutation $\begin{pmatrix}1&2&3&4&5&6\\5&6&2&4&1&3\end{pmatrix}$ *is odd and* $\begin{pmatrix}1&2&3&4&5&6\\6&3&4&5&2&1\end{pmatrix}$ *is even*

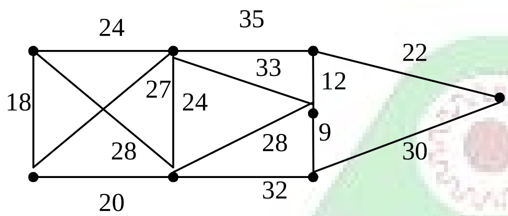
36. a. Let $X=\{1,2,3,4,5,6,7\}$ and $R=\{(x,y)/(x-y)\text{ is divisible by }3\}$ .Show that *R* will form an equivalence class, find them also draw the graph of *R*

b. Let $A=\{1,2,3,4\}$ *and* $B=\{a,b,c\}$. Let $R=\{(1,a),(1,b),(2,b),(2,c),(3,b),(4,a)\}$ $S=\{(1,b),(2,c),(3,b),(4,b)\}$ .Compute $\overline{R}, R\cap S$ *and* R^{-1}

37. a. Draw the logic diagram to represent the Boolean expression

$$p(x,y)=\left((x'\wedge y)'\vee(y\wedge x)\right).$$

b. Find the minimal spanning tree for the following graph



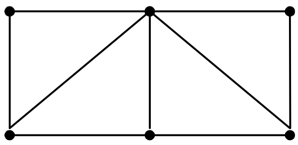
38. a. Use Karnaugh map simplify the Boolean expression
$f(a,b,c,d)=a'b'cd+a'b'cd+a'bc'd+ab'c'd'+ab'c'd+ab'cd+abc'd'+abcd'+abcd$

b. Give the tree representation of the expression $((x+5)*[(6*y+z)/(x+4)])$ and find its value for $x=4, y=1, z=2$

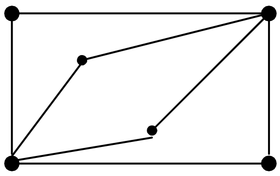
39. a. State and prove the necessary and sufficient condition for a connected graph G is to be an Euler graph

b. Prove that a group G is abelian if and if only $(ab)^2=a^2b^2$

40. a. Determine whether the following graphs are Hamiltonian or Eulerian or both



(1)

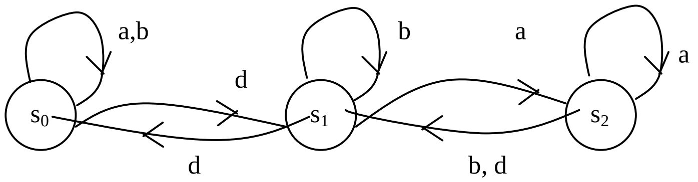


(2)

b. Determine the multiplication table of the quotient $Z/3Z$, where Z has the operation +

39. a. Find the phase-structure grammar to generate the language $L(G)=\{a^nb^nc^n/n\geq 1\}$

b. Let $S=\{s_0,s_1,s_2\}, I=\{a,b,d\}$, and consider the finite-state machine $M=(S,I,F)$ defined by the digraph verify $f_{abd}\circ f_{bad} = f_{badadd}$



40. a. Determine the group code $e_H : B^2 \to B^5$ where $H=\begin{bmatrix}1&1&0\\0&1&1\\1&0&1\\0&1&0\\0&0&1\end{bmatrix}$

b. Construct a Finite State machine which accepts those input sequence *w* that contain the string 01 or the string 10 anywhere with in them

41. a. Find the truth value of each statement if p and q are true and r, s are false:
(i)(~ q) $\Rightarrow$ (r$\Rightarrow$ (r $\Rightarrow$ (p V s))) (ii) p $\Rightarrow$ (r $\Rightarrow$ q)

b. By mathematical induction, prove that 1+2+3+…+n = [n(n+1)]/2, $\forall n \geq 1$.

42. a. Solve the recurrence relation $f_n = f_{n-1} + f_{n-2}$, n ≥ 2 with initial conditions $f_0 = 0$, f_1 =1.

b. Let $A=\begin{bmatrix}1&0&1\\0&1&1\\1&1&0\\0&0&0\end{bmatrix}$, $B=\begin{bmatrix}1&1&0\\1&0&1\\0&0&1\\1&1&0\end{bmatrix}$ Compute $A \vee B$ *and* $A \wedge B$.

43. a. Let A = {a, b, c, d} and R be the relation on A that has the matrix.
$M_R=\begin{bmatrix}1&0&0&0\\0&1&0&0\\1&1&1&0\\0&1&0&1\end{bmatrix}$. Construct the diagraph of R and list in degrees and out degrees of all vertices.

b. Let $A=\{1,2,3,4\}$, $R=\{(1,2),(2,3),(3,4),(2,1)\}$. Find the transitive closure of *R*.

44. a. Let $A=\{1,2,3,4\}$ *and* $B=\{a,b,c\}$. Let $R=\{(1,a),(1,b),(2,b),(2,c),(3,b),(4,a)\}$
$S=\{(1,b),(2,c),(3,b),(4,b)\}$.Compute $\overline{R}, R\cap S, R\cup S$ *and* S^{-1}

b. If A and B are real numbers and f is defined on A by f (x) = $2x^3 - 1$ and g is given by $g(y)=\sqrt[3]{\frac{y+1}{2}}$. Show that f is a bijection between A and B and g is bijection between B and A.

45. a. Let S = {a, b, c}, A= P(S), Draw the Hasse diagram for the poset A with the partial order$\subseteq$.

b. In Boolean algebra, state and prove the De Morgan's laws.

46. a. Let $p(x,y,z)=((x\wedge y)\vee(y\wedge z'))$. Give the truth table for the corresponding

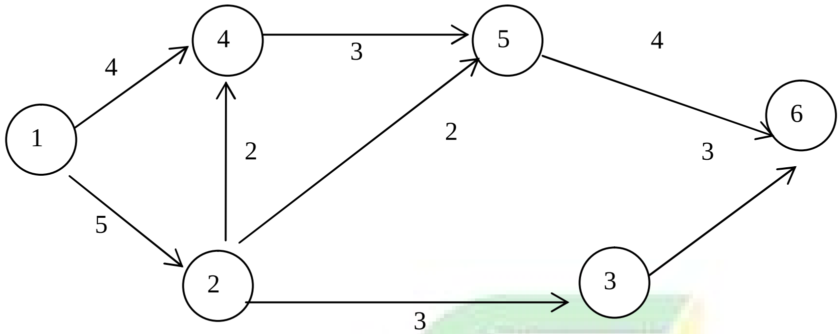
function $f: B_3 \to B$. Also draw the logic diagram for p.

b. Obtain the tree for the algebraic expression
(3 × (1 – x)) ÷ (4 +((7 – (y–2)) × (7 + (x ÷y)))).

47. a. Explain: Euler path, Euler circuit, Hamiltonian path and Hamiltonian circuit with illustrations.

b. Let G be the set of all nonzero real numbers and let $a * b = \frac{ab}{2}$.Show that $(G,*)$ is an abelian group.

48. Find a maximum flow in the given network by using Labelling Algorithm.



49. a. Explain types of grammars with illustrations.

. b. Let H = $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ be a parity check matrix. Determine the (2, 5) group code $e_H : B^2 \to B^5$

50. a. Draw the diagraph of the machine whose state transition table is given below.

	0	1	2
s_0	s_1	s_0	s_2
s_1	s_0	s_0	s_1
s_2	s_2	s_0	s_2

b. Let e: $B^m \to B^n$ be a group code. Prove that the minimum distance of e is the minimum weight of a non zero code word.

51. a. For any 3 sets A, B, C, Prove that A ∪ (B ∪ C) = (A ∪ B) ∩ (A ∪ C).
b. Using Euclidean algorithm, find the G.C.D of (190, 34) and express it as d = sa + tb.

52. a. Prove that (q ∧ (p → q)) → P is a tautology.
b. Solve the recurrence relation $a_n = a_{n-1} + 2a_{n-2}$; n ≥ 2 with initial conditions $a_1 = 0, a_2 = 2$. (7)

53. a. Let $A = \{a_1, a_2, a_3, a_4, a_5\}$ & $R = \{(a_1, a_1), (a_1, a_4), (a_2, a_2), (a_2, a_3), (a_3, a_4), (a_3, a_5), (a_4, a_1), (a_5, a_2), (a_5, a_5)\}$. Compute w_1, w_2, w_3 using Warshall Algorithm.

    b. Let A = {1, 2, 3, 4} & R = {(a, b) / a – b is divisible by 3} Prove that R is an equivalence relation and find R.

54. a. Let A = B = C = R and consider the function f: A → B & g : B→C defined by f(a) = 2a + 1 & $g(b) = \frac{b}{3}$. Verify $(gof)^{-1} = f^{-1} o\, g^{-1}$.

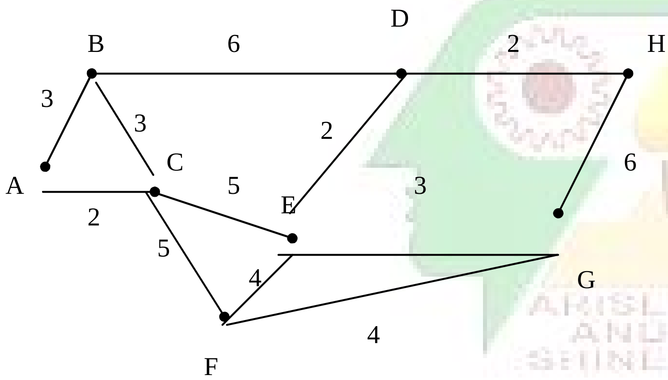
    b. If $p\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 2 & 4 & 5 & 7 & 6 & 3 & 1 \end{pmatrix}$ (i) write p as a product of disjoint cycles (ii) Is 'p' even or odd? (iii) compute p^2.

55. a. Show that D_{30} is a Boolean Algebra.

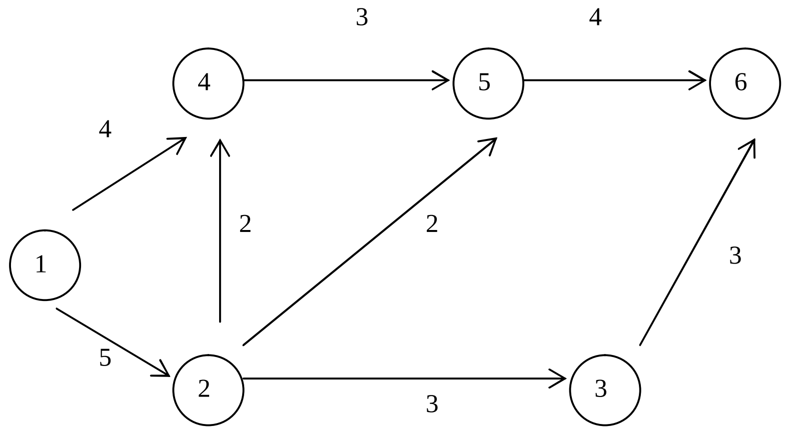
    b. Construct the k-map for the function 'f' and find the Boolean expression. s(f) = {(0,0,0) , (0,0,1) , (0,1,1) , (1,0,0) , (1,1,1)}.

56. a. Draw a binary tree whose post order search produces the string
    (i) SEARCHING
    (ii) TREE HOUSE

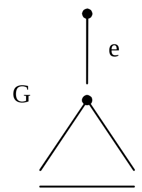
b. Find the minimal spanning tree for the graph given below.



57. Find a maximum flow in the given network by using Labelling Algorithm.



58. a. Find $P_G(x)$ and ψ (G) for the following graph, using edge 'e'.



b. If f is a homomorphism from a commutative then semigroup (s, ∗) on to a semigroup (T, ∗') , then prove that (T, ∗') is also commutative.

59. a. Let M = (S, I, F) be a finite state machine, whose transition table is given by

	0	1
s_0	s_0	s_1
s_1	s_2	s_2
s_2	s_1	s_0

List the values of the transition functions f_w for w = 011.

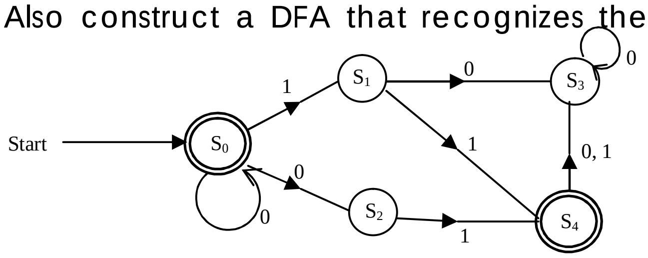
b. Construct the phrase structured grammar G for the following L(G).
   i. L (G) = $\{a^n b^n / n \geq 1\}$
   ii. L (G) {strings of 0's and 1's with an equal number}

60. a. Explain briefly, the types of grammar with example.

b. Let $H = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ be a parity check matrix. Determine the (2, 5) group code function $e_H : B^2 \to B^5$.

61. a. Define context free grammar, find a context free grammar that generates the set of all palindromes over {0,1}. Check also the acceptance of the input string 110011.

b. Find the Language recognized by NDFSA, whose state diagram is given below. Also construct a DFA that recognizes the same Language as the NDFSA.



62. a. Show that $\neg p \to (q \to r)$ and $q \to (p \vee r)$ are Logically equivalent.

b. Show that the hypotheses "It is not sunny this afternoon and it is colder than yesterday" "we will go swimming only if it is sunny", "If we do not go swimming, then we will take a canoe trip" and "If we take a canoe trip, then we will be home by sunset" lead to the conclusion "We will be home by sunset".

63. a. If f : A→B and g : B→C are bijections, prove that gof : A→C is also bijection.

b. Find the sum-of-products expansion for the function $F(x,y,z) = (x+y)\bar{z}$ using Boolean identities and truth table. Also construct a logic gate diagram for sum-of-products form.

64. a. Prove, using mathematical induction, that $1.2 + 2.3 + \ldots\ldots + n(n+1) = \frac{n(n+1+(n+2)}{3}$

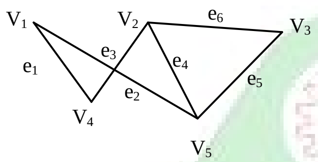
b. The relations R and S on a set A are represented by the matrices $M_R = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \end{pmatrix}$ and $M_S = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$. Find the matrices that represent

(i) $R \cup S$ (ii) $R \cap S$ (iii) SoR (iv) RoR and (v) R^{-1}.

65. a. Define equivalence relation, Show that the relation $R = \{(a,b) / a \equiv b \pmod m\}$ is an equivalence relation on the set of integers where $m > 1$.

b. Prove $(z^+, /)$ is a poset. Find the greatest lower bound and the least upper bound of the set {3,9,12} in the poset $(z^+, /)$.

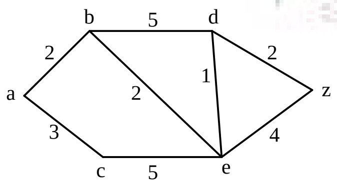
66. a. Define Adjacency matrix and incidence matrix of an undirected graph. Find the incidence matrix and adjacency matrix for the graph shown below.



b. Explain connectedness in directed graphs with examples.

67. a. Define planar graph. Is the complete graph K_4 planar? Prove that if G is a connected planar simple graph, then G has a vertex of degree not exceeding five.

b. Find a shortest path between a and z in the following weighted graph.

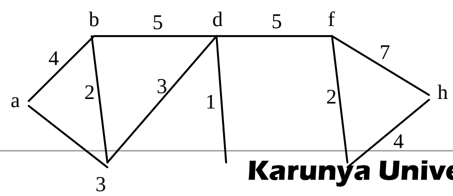


68. a. Prove that an undirected graph is a tree if and only if there is a unique simple path between any two of its vertices.

b. Find the ordered rooted tree representing the compound proposition $(\neg(p \wedge q)) \leftrightarrow (\neg p \vee \neg q)$. Also find the prefix, postfix and infix forms of this expression.

69. a. Prove that a tree with n vertices has (n-1) edges.

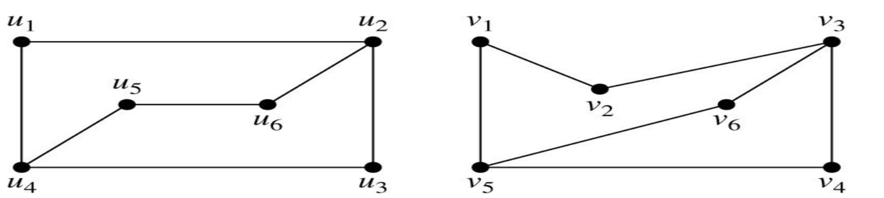
b. Explain Kruskal's algorithm to find a minimum spanning tree for connected weighted graph. Using the algorithm, construct a minimum spanning tree for the following graph.



70. a. What is the value of the prefix expression + - * 2 3 5 / ↑ 2 3 4?
    b. Using alphabetical order, construct a binary search tree for the words in the sentence, "The quick brown fox jumps over the lazy dog".
    c. Prove: A tree with n vertices has n – 1 edges.
    d. Construct the tree of the Algebraic expression ((A x B) + (C – (D x E))) + (F – (G x H)).
72. a. Show that the following statement is inconsistent: If Jack misses many classes through illness, then he fails high school. If Jack fails high school, then he is uneducated. If Jack reads a lot of books then he is not uneducated. Jack misses many classes through illness and reads a lot of books.
    b. Symbolize the expression "All the world loves a lover".
    c. S.T from (a) $(\exists x)[F(x)\wedge S(x)]\rightarrow(\forall y)[M(y)\rightarrow W(y)]$ (b)$(\exists y)[M(y)\wedge\neg W(y)]$ the conclusion $(\forall x)[F(x)\rightarrow \neg S(x)]$ follows.
73. a. Construct the Truth Table and Circuit Diagram for the expression
    F (x, y, z) = $xyz + x\overline{y}\overline{z} + \overline{x}y\overline{z} + \overline{x}\overline{y}z$.
    b. Draw a Karnaugh map for a function in three variables. And simplify the following usingKarnaugh map $f(x_1,x_2,x_3,x_4)=x_1x_3+x_1'x_3x_4+x_2x_3'x_4+x_2'x_3x_4$
74. a. Use Mathematical Induction to prove that $8^n - 3^n$ is multiple of 5 whenever n is a positive integer.
    b. Define: Ackermann's function A (m, n) inductively and find the value A (2, 2).
    c. Verify that the program segment is correct with respect to the initial assertion Y = 3 and the final assertion z = 6.

```
X: = 2
Z: = X + Y
If Y > 0 then
Z: = Z + 1
Else   Z: = 0
```

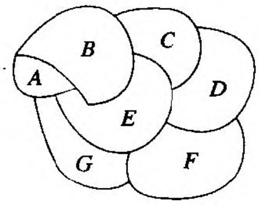
75. a. Prove that the relation "congruence modulo m" over the set of positive integers is an equivalence relation.
    b. Given A = {1, 2, 3, 4} and R = {(1, 2) (1, 1) (1, 3) (2, 4) (3, 2)};
    S = {(1, 4) (1, 3) (2, 3) (3, 1) (4, 1)} are relations on A, compute SoR, RoS, RUS, R ∩ S also represent them by Digraph and Matrix.
    c. Draw the Hasse Diagram for the partial ordering set (D (128), ≤) and ≤ is the relation "is a Divisor of".
76. a. A simple graph with n vertices and k components can have at most (n – k) (n – k + 1) / 2 edges – Prove.
    b. Determine whether the graphs G and H are Isomorphic.



G *H*

77. a. A connected planar graph with v – vertices and e – edges has e – v + 2 = r, the number of regions- Prove.

b. Define Chromatic number. What is the chromatic number of K_n and C_n?
c. Construct the dual graph of the following and find its Chromatic number.



78. a. Construct a Phrase – structure grammar that generates the set $\{0^n1^n / n \geq 0\}$.
b. Construct a non-deterministic finite – state automaton that recognizes the regular set 01* U 00*1.
c. Let G = {V, T, P, S} where V = {a, b, c, S}, T = {a, b, c}, S is the start symbol and P = {S→abS / bcS / bbS / a / cb}. Derive the Strings bcbba, bbbcbba, bcabbbbbcb.
79. a. Construct a Turing Machine for adding two nonnegative integers.
b. Find a Turing Machine that recognizes the set $\{0^n1^n / n \geq 1\}$.
c. Briefly Explain the Church-Turing Thesis.

c 6 e 5 g

80. a. Out of 270 residences in Chennai, 64 were found to have piped water supply, 94 were supplied by tankers and 58 had their own wells. If 26 had piped water and were supplied by tankers. 28 piped water and wells, 22 had wells and water supplied by tankers and 14 had all the three sources of water, how many of the houses did not depend on any of these sources? How many of them depend only on one of these sources?
b. Show that the following formula is a tautology. $(P \Leftrightarrow Q) \Leftrightarrow (P \wedge Q) \vee (\sim P \wedge \sim Q)$.
81. a. In how many ways can a committee of 8 is to be formed from 10 women and 12 men, if the committee should contain
(i) equal number of men and women.
(ii) at least 3 women.
b. Solve the recurrence relation $d_n = 2d_n - d_{n-1}$ with initial conditions $d_1 = 1.5$ and $d_2 = 3$.
82. a. If A denotes the set of divisors of 24 and R the relation on A given by aRb if a divides b, find A, R, the matrix of R.
b. If $f : Z \rightarrow Z$ and $g: Z \rightarrow Z$ are functions defined on the set of integers Z by f(x) = 3x, g(x) = x + 5, find fog, gof and g^{-1}.
83. a. If $M_R = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ and $M_S = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}$ are the matrices of R and S respectively find M_{SoR} and hence determine if SoR is (i) reflexive (ii) symmetric.
b. If $f : A \rightarrow B$ and $g : B \rightarrow C$ are bijective show that $(gof)^{-1} = f^{-1}og^{-1}$.
84. a. Define a tree. Show that a tree with n vertices has n-1 edges.
b. Show that (P(A), ⊆) is a poset where P(A) is the power set of a set A = {a, b, c} and ' ⊆ ' means 'is a subset of'. Draw the Hasse diagram of P(A)
85. a. Show that $f : Z^+ \rightarrow 2Z^+$ given by f(x) = 2x is an isomorphism from the semigroup $(Z^+, \leq)$ to the semigroup $(2Z^+, \leq)$ where Z^+ denotes the set of positivintegers, $2Z^+$ denotes the set of positive even integers and ' ≤ ' means less than.

b. Show that if (T, v_0) is a rooted tree then T has no cycles

86. a. If a, b ϵ G where G is a group, show that the equation ax = b has an unique solution in G

b. Show that a connected graph with exactly 2 vertices of odd degree has an Euler path.

87. a. Show that every element of a group has an unique inverse.

b. Show that in a graph if every vertex is of even degree then the graph has an Euler circuit

88. a. Explain Backus Naur form. Rewrite as Backus Naur form $G = \{V, S, v_0, \mapsto\}$, $V = \{ v_0, v_1, a, b\}$, $S = \{ a, b\}$, $\mapsto : v_0 \mapsto av_1, v_1 \mapsto bv_0 , v_1 \mapsto a$

b. Show that the minimum distance of a group code is the minimum weight of a nonzero code word.

89. a. What is the language described by $G = \{V, S, v_0, \mapsto\}$, $V = \{v_0, v_1, x, y, z\}$, $S = \{ x, y, z\}$, $\mapsto : v_0 \mapsto xv_0, v_0 \mapsto yv_1, v_1 \mapsto y v_1, v_1 \mapsto z$

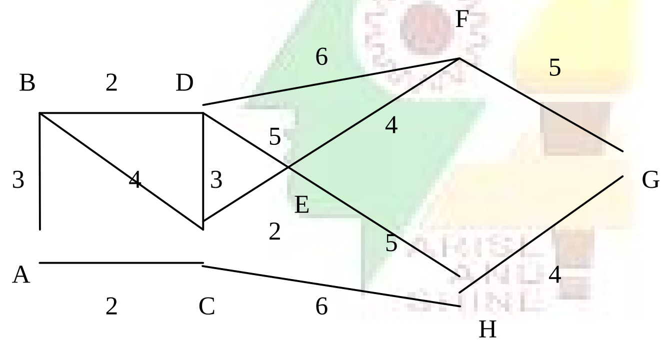
b. If K is a finite subset of a group G show that every left coset of K in G has the same number of elements as K.

90. Obtain the tree for the algebraic expression

a. $(3 \times (1 - x)) \div ((4 + (7 - (y\_2)) \times (7 + (x \div y))))$

Also write down the pre order and post order traversals of the tree.

b. Using Kruskal's algorithm, find the minimal spanning tree for the given graph.



91. Find a maximum flow in the given network by using labeling Algorithm.

92. a. Solve the Fibonacci recurrence relation $f_n = f_{n-1} + f_{n-2}$, $f_1 = f_2 = 1$.

b. Prove by Induction $n < 2^n$ for n>1.

93. a. A survey has been taken on methods of commuter travel. Each respondent was asked to check BUS, TRAIN or AUTOMOBILE as a major method of traveling to work. More than one answer was permitted. The results reported were as follows: BUS, 30 people; TRAIN, 35 people; AUTOMOBILE, 100 people; BUS and TRAIN, 15 people; BUS and AUTOMOBILE,15 people; TRAIN and AUTOMOBILE, 20 people and all 3 methods, 5 people. How many people completed the survey form?

b. In how many ways can six men and six women be seated in a row if any person may sit next to any other? How many ways are there if men and women must occupy alternate seats?

94. a. Consider the relation defined from A={1,2,3,4} to B={1,4,6,8,9} as, aRb iff $b = a^2$. Find the relation R and represent it using a digraph. Also give the matrix of R.

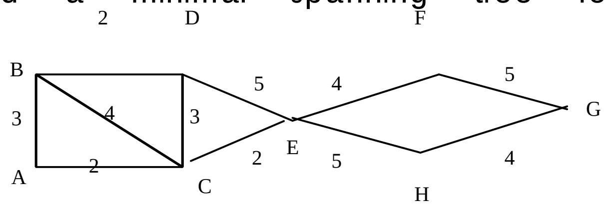
b. Use Warshall's algorithm to find the transitive closure of R = {(1,1), (1,4),(2,1), (2,3), (3,1), (3,2), (3,4), (4,2)} on the set A = {1,2,3,4}.

95. a. Prove that the relation aRb iff $a \equiv b \pmod 4$ defined on Z is an equivalence relation.

   b. Let A={2,3,6,12}and let R and S be the following relations on A. xRy iff 2 / x–y. xSy iff 3 / x–y. Compute the following $\overline{R}, R \cap S, R \cup S, S^{-1}$.

96. a. Draw the Hasse diagram for divisibility relation on the set {1, 2, 3, 6, 12, 24, 36, 48}. What are the minimal and maximal elements?

   b. Find a minimal spanning tree for the graph given below.

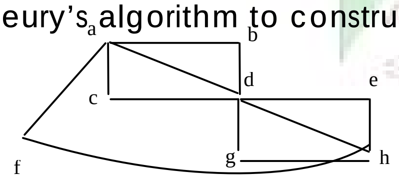


97. a. Which of the following are Boolean Algebras? D_{210}, D_{66}, D_{30}, D_{25}.

   b. Given the following truth table of a function f, construct the corresponding K-map and hence find an expression for f.

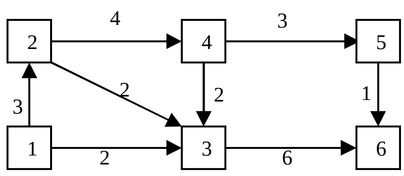
x	y	z	f(x,y,z)
0	0	0	1
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

98. a. Use Fleury's algorithm to construct an Euler circuit for the graph given below.



   b. Let (S,*) and (T,*') be monoids with identities e and e', respectively. Let f: S $\rightarrow$ T be an isomorphism. Then show that f(e) = e'.

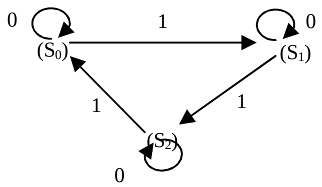
99. a. Find a maximum flow in the given network.



   b. Prove that if G is a graph with m number of edges then G has a Hamiltonian circuit if $m \geq \frac{1}{2}(n^2 - 3n + 6)$.

100. a. Consider the machine whose graph is shown below. Show that $f_w(s_0) = s_0$

iff w has 3n 1s for some $n \geq 0$.



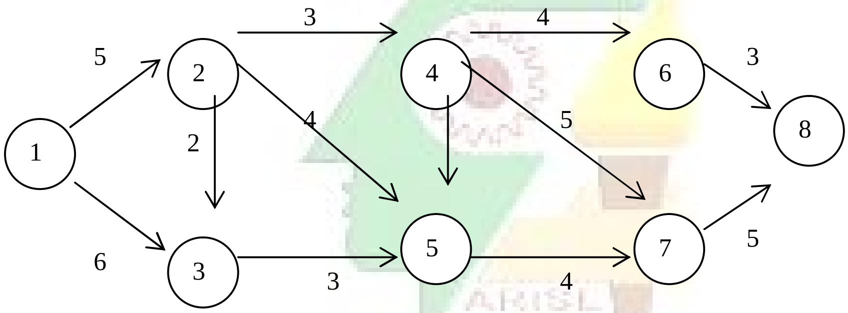
b. Consider the (3,8) encoding function $e : B^3 \rightarrow B^8$ defined by

e(000) = 00000000 e(001) = 10111000
e(010) = 00101101 e(011) = 10010101
e(100) = 10100100 e(101) = 10001001
e(110) = 00011100 e(111) = 00110001

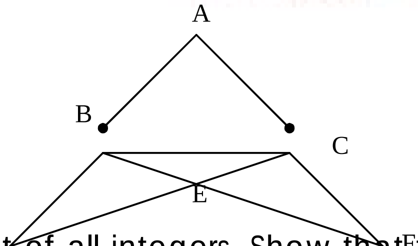
How many errors will e detect?

101. a. Let H = $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ be a parity check matrix. Determine the (2,5) group code function $e_H : B^2 \rightarrow B^5$.

b. Let M=(S,I,F) be a finite state machine. If w_1 and w_2 are in I^* then prove that $T(w_1.w_2)=T(w_2)\circ T(w_1)$.Also prove that if $M = T(I^*)$, then M is a submonoid of S^S.



102. a. Apply Fleury's Algorithm and construct an Euler circuit for the given graph.



b. Let T be the set of all integers. Show that the semi groups (z, +) and (T, +) are isomorphic.

103. Consider the machine whose transition table is

	0	1
1	1	4
2	3	2
3	2	3
4	4	1

Here S = {1, 2, 3, 4}

i. S.T R= {(1,1), (1,4) , (4, 1), (4, 4) (2, 2) , (2, 3) , (3, 2) , (3, 3)} is a machine congruence.

ii. Construct the state transition table for the corresponding quotient machine and

iii. Construct the digraph of the quotient machine.

104. a. Let H = $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ be a parity check matrix. Determine the (2, 5) group code function. $e_H : B^2 \rightarrow B^5$.

b. Construct a phrase structure grammar G such that the language L(G) of G is equal to the language L.

i. L = $\{a^n b^n / n \geq 3\}$

ii. L = $\{a^i b^{2i} / i \geq 1\}$

105. a. Show that $\sqrt{3}$ is Irrational

b. Construct the truth table for $[(P \vee Q) \wedge (P \Rightarrow R) \wedge (Q \Rightarrow R)] \Rightarrow R$

106. a. Prove that the following statement is true using Mathematical Induction $1^2 + 2^2 + 3^2 + \ldots\ldots\ldots + n^2 = n (n + 1) (2n +1) / 6$

b. Find an Explicit formula for the Fibonacci numbers.

107. a. Let X = {1, 2, 3, 4, 5, 6, 7} and R = {< x, y > | x-y is divisible by 3}. S.T R is an equivalence relation.

b. Suppose that the Relations R1 and R2 on a set A are represented by the matrices $M_{R1} = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$ and $M_{R2} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}$. What are the matrices and Digraph representation of R1 U R2, R1 ∩ R2?

108. a. Show that f and g have the same order if $f(n) = 3n^4 - 5n^2$ and $g(n) = n^4$ defined for positive integers.

b. In a Cyclic Permutation on A = {1, 2, 3, 4, 5, 6}, Compute the following: (4, 1, 3, 5) o (5, 6, 3), (5, 6, 3) o (4, 1, 3, 5)

109. a. Prove: If a complement exists in a distributed lattice, it is Unique.

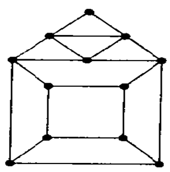
b. Prove that if $a \leq c$ and $b \leq d$, then $a \vee b \leq c \vee d$; $a \wedge b \leq c \wedge d$

110. a. Construct the truth table and Logic diagram for the expression $p(x, y, z) = (x \wedge y) \vee (y \wedge \sim z)$.

b. Construct the tree of the Algebraic expression ((2 x A) + (3 – (4 x A))) + (A – (3 x 11))

111. Define Hamiltonian Circuit. Prove that the Graph G with n – vertices and m – edges has Hamiltonian circuit if $m \geq \frac{1}{2}(n^2 - 3n + 6)$.

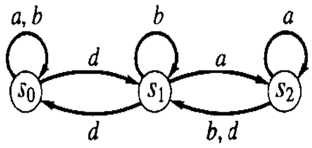
112. a. Define Chromatic Number. Find the Chromatic number of the Graph.



b. Let G be the set of all nonzero real numbers and let a*b = ab / 2. Show that (G, *) is an Abelian group.

113. a. Classify the Types of Phrase-Structure Grammar.

b. Compute f_{bad}, f_{dad} and f_{badadd} from the finite state machine (S, I, f), S = {s_0, s_1, s_2}, I = {a, b, d} defined by the digraph



114. Find the code words generated by the parity check matrix when the coding function is e: $B^3 \rightarrow B^6$ $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

114. a. Compute GCD (273, 98) using Euclidean Algorithm.

b. Define $A \vee B$ and $A \wedge B$ for any two matrices A and B.

c. Find $A \vee B$ and $A \wedge B$ if $A = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$

115. a. Prove that $\sim (p \Rightarrow q) \Rightarrow p$ is a tautology.

b. Prove that 3 divides (n^3-n) for every positive integer n.

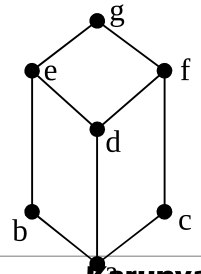
116. a. Let L be the set of straight lines in the plane. For any two lines l_1 and l_2 we define l_1 R l_2 if l_1 is perpendicular to l_2. Check whether this defines an equivalence relation on L.

b. If A = {1, 2, 3, 4} and R = {(1,2), (2,3), (3,4), (2,1)}. Find the transitive closure of R by drawing the digraph of R and finding R^∞.

117. a. Let A = B = R, the set of real numbers. Let $f : A \rightarrow B$ be given by the formula f(x)=2x^3-1 and let g be given by $g(y) = \sqrt[3]{\frac{1}{2}y + \frac{1}{2}}$. Show that f is a bijection between B and A.

b. Write the permutation $p = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 3 & 4 & 5 & 5 & 2 & 1 & 8 & 7 \end{pmatrix}$ as a product of transpositions.

118. a. Find out whether the following is a lattice (by finding out LUB and GLB of every pair of elements) and if whether it is distributive or not.

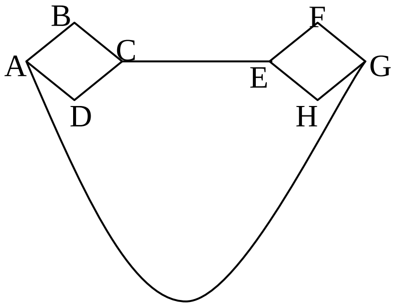


b. Find all divisors of 36 and draw the lattice D_{36}, the lattice of all divisors of 36 where a≤b if and only if a divides b. Find whether this lattice is complemented.

119. a. Let A = {q0, q1, q2, q3, q4, q5, q6, q7, q8, q9} and let T = {(q1,q2), (q1,q0), (q3, q4), (q3,q5), (q4,q7), (q5,q6), (q3,q1), (q6,q8), (q6,q9)}. Show that T is a rooted tree and identify the root.

b. Draw the labeled tree for the expression (3-(2*x))+((x-2)-(3+x)).

120. Use Fluery's algorithm to construct an Euler circuit for the following graph.



121. a. Let (S,*) and (S,*') be monoids with identities e and e' respectively. Let $f : S \to T$ be an isomorphism. Then prove that f(e) = e'.

b. Let S = {1, 2, 3, 6, 12}. The operation * in S is defined as a*b = gcd(a,b). Find whether S is a monoid and if so find the identity element.

122. Let V = {v_0,w,a,b,c}, S = {a,b,c} and let → be a relation on V* given by $v_0 \to av_0b$, $v_0b \to bw$, $abw \to c$. Explain the process of finding L(G) and give the derivation of three different allowable sentences.

123. Consider the (3, 8) encoding function $e: B^3 \to B^8$ defined by the code words

e(000) = 00000000
e(001) = 10111000
e(010) = 00101101
e(011) = 10010101
e(100) = 10100100
e(101) = 10001001
e(110) = 00011100
e(111) = 00110001 How many errors will e detect?

124. a. Give an example of a lattice where the distributive laws do not hold.

b. Find a Boolean polynomial p that induces the function f.

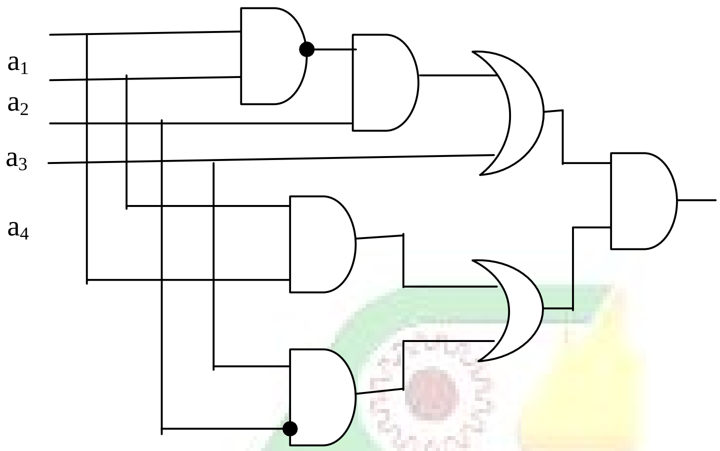
b_1	b_2	b_3	$f(b_1,b_2,b_3)$
0	0	0	1←
0	0	1	0
0	1	0	0
0	1	1	1←
1	0	0	1←
1	0	1	0
1	1	0	0
1	1	1	0

c. Draw Hasse Diagram for (D_{30}, '/').

125. a. Find the minterm normal form for the expression $f(x_1,x_2,x_3)=((x_1+x_2)+(x_1+x_3)).x_1.x_2'$
b. State and prove Representation theorem.

126. a. Explain some special gates with its diagram.
b. Explain Half-adder.

127. a. Determine (i) the symbolic representation of the circuit given by $p=(x_1'x_2)'+x_3$. (ii) Determine the Boolean polynomial p of the following circuit and simplify p.



b. Describe one application of switching circuit with an example.

128. a. Factorize $x^{15}-1$ over F_2.
b. State and prove Wilson's Theorem.

129. a. Determine the elements of $F_2{}^3$.
b. Define Mobius Function. State and Prove Mobius inversion formula.

130. Prove that a polynomial $f \in F_q[x]$ of degree m is primitive if and only if f is monic, $f(0) \neq 0$, and the order of f is equal to q^m-1.

131. Explain Berlekamp's Algorithm and determine the complete factorization of $g=x^8+x^6+2x^4+2x^3+3x^2+2x$ over F_5.

132. a. Define linear code, Parity-check matrix, Systematic Linear code, Group code, Repetition code. Give examples for each one.
b. Let C be an ideal $\neq \{0\}$ of V_n. Then Prove that there exist a unique $g \varepsilon V_n$ with the following properties: (i) $g \mid x^n - 1 \mid$ in $F_q[x]$; (ii) $C=(g)$; (iii) g is monic
c. Prove that the following statement A linear code $C \subseteq V_n$ is cyclic if and only if C is an ideal in V_n.

133. a. Construct a Turing machine for adding 2 non-negative integers.
b. Define Finite State machine, 0-type, type-1, type -2 and type-3 grammar.
c. Let G be the grammar with V = (S, A, a, b), set of terminals T = (a,b), starting symbol S and productions P = ($S \to aA$, $S \to b$, $A \to aa$). What is L (G), the language of this grammar?

134. a. Show that $(p \wedge q) \to (p \vee q)$ is a tautology.
b. Let f and g be the function from the set of integers to the set of integers

defined by f(x) =2x + 3 and g(x) = 3x + 2. What is the composition of f and g? What is the composition of g and f.

c. Use Karnaugh map to simplify the sum of products expansions $WX\overline{Y}\overline{Z}+W\overline{X}YZ+W\overline{X}Y\overline{Z}+W\overline{X}\overline{Y}\overline{Z}+\overline{W}X\overline{Y}\overline{Z}+\overline{W}\overline{X}Y\overline{Z}+\overline{W}\overline{X}\overline{Y}\overline{Z}$.

135. a. Show that the distributor law x (y + z) = xy + xz is valid

b. Find the sum of products expansion for the function F(x, y, z) = (x + y) $\overline{z}$ using Boolean identities.

c. What are the contra positive, converse and the inverse of the implication: The home team wins wherever it is raining.

136. a. Use Mathematical induction to prove $n^3 - n$ is divisible by 3 whenever n is a positive integer.

b. Show that the inclusion relation $\subseteq$ is a partial ordering on the power set of a set S.

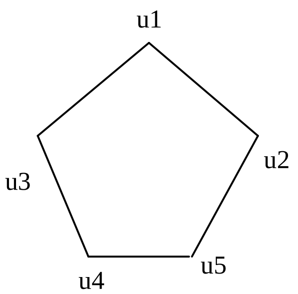
c. Find the matrix representing the relations SoR where the matrix representing R and S are $M_R = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ $and\ M_S = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix}$

137. a. Let m be a +ve integer with m > 1. Show that the relation R = (a, b) / a $\equiv$ b (modm) is an equivalence relation on the set of integers.

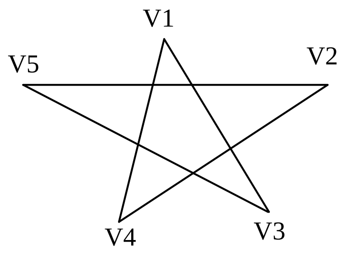
b. Find the Zero – one matrix of the transitive closure of the relation R where $M_R = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$.

c. Prove: If a and r are real numbers r $\neq$ 0 then $\sum_{j=0}^{n} ar^j = \begin{cases} \frac{ar^{n+1}-a}{r-1} & if\ r \neq 1 \\ (n+1)a & if\ r = 1 \end{cases}$

138. a. Prove that the graph G and $\overline{G}$ given below are isomorphic.



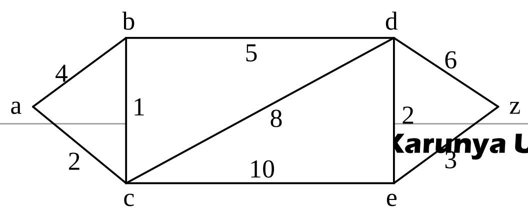
G



$\overline{G}$

b. An undirected graph has an even number of vertices of odd degree.

139. a. Find the shortest path from a to z in the weighted graph.

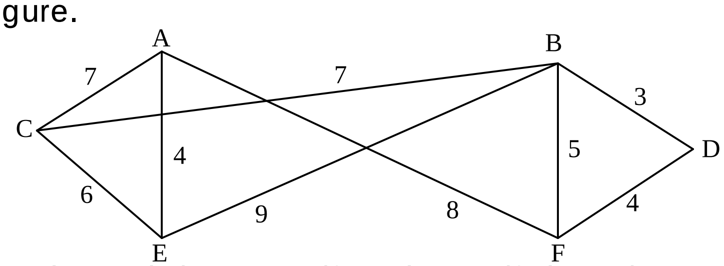


b. Let G be a connected planar simple graph with *e* edges and V vertices. Let r be the number of regions in a planar representation of G then r = *e*–v +2.

140. a. What is the value of the post fix expression 723* − 4 ↑ 93 / +?

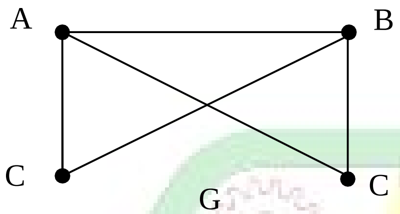
b. Prove that a tree with n vertices has (n-1) edges.

141. a. Find a minimum spanning tree of the labeled connected graph shown in the given figure.



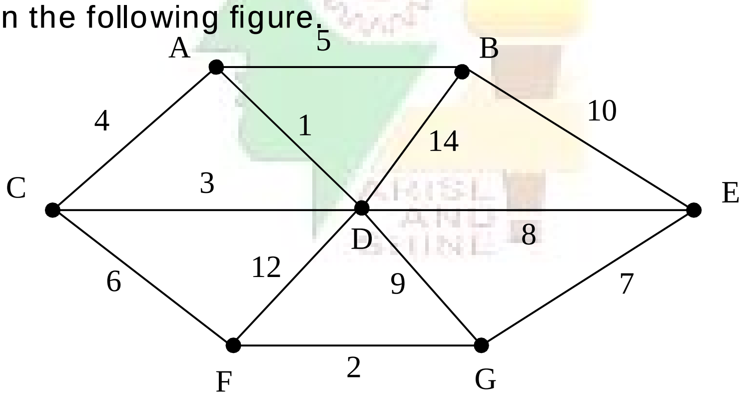
b. An undirected graph is a tree if and only if there is a unique simple path between any 2 of its vertices.

142. a. Draw all the spanning trees of the graph G shown is the following figure.



G

b. Use Prim's algorithm to find a minimum spanning tree for the weighted graph in the following figure.



143. a. Prove that $\forall x(P(x) \to (Q(y) \wedge R(x)))$, $\exists\, xP(x) \Rightarrow Q(y) \wedge \exists x(P(x) \wedge R(x))$.

b. Simplify the Boolean expressions given below:

i. $(x + y + xy)(x + z)$

ii. $x[y + z(zy + xz)']$

144. a. Give a direct proof for the implication $p \to (q \to s), (\neg r \vee p),\ q \Rightarrow (r \to s)$.

b. If S= {1, 2, 3, 4, 5} and if the function f, g, h: S→S are given by

f = {(1,2), (2,1), (3,4), (4,5), (5,3)}

g = {(1,3), (2,5), (3,1), (4,2), (5,4)}

h = {(1,2), (2,2) (3,4) (4,3), (5,1)}

i. Verify whether $f \bullet g = g \bullet f$

ii. Find f^{-1} and g^{-1}.

iii. Find $(f \bullet g)^{-1}$ and show that $(f \bullet g)^{-1} = g^{-1} \bullet f^{-1} \neq f^{-1} \bullet g^{-1}$

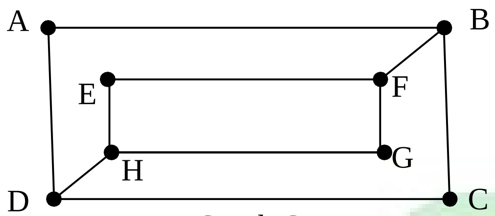
145. a. Prove, by mathematical induction, that $1^2+3^2+5^2+\ldots\ldots.+(2n-1)^2=\frac{1}{3}n(2n-1)(2n+1)$.

b. If R is the relation on the set of the integers such that (a, b) $\in$R, if and only if 3a+4b =7n for some integer ‘n’, prove that R is an equivalence relation.

146. a. Use mathematical induction to show that $\frac{1}{\sqrt{1}}+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{3}}+\ldots\ldots\ldots\ldots+\frac{1}{\sqrt{n}}>\sqrt{n},\ for\ n\geq 2,$

b. Draw the diagraph representing the partial ordering {(a,b)| a divides b} on the set {1, 2, 3, 4, 5, 6, 7, 8}. Reduce it to the Hasse diagram representing the given partial ordering.

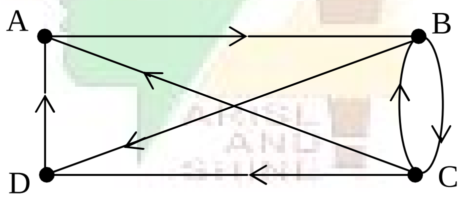
147. a. Determine whether the graphs shown in the following figures are isomorphic.



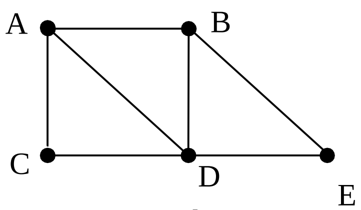
Graph G_1

Graph G_2

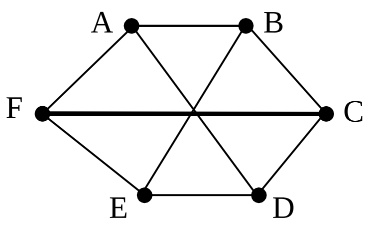
b. Find the number of paths of length 4 length from the vertex B to the vertex D in the directed graph shown in the following figure, analytically. Name those paths using the graph.



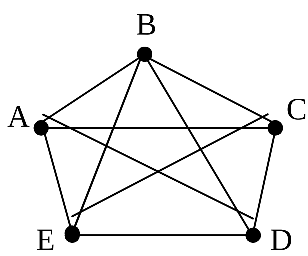
148. a. Find an Euler path or an Euler circuit, if it exists in each of the three graphs is the following figures. If it does not exist, explain why?



Graph G_1



Graph G_2



Graph G_3

b. Prove that the number of edges in a bipartite graph with n vertices is at most $\left(\frac{n^2}{2}\right)$.

149. a. Find the language generated by each of the following grammars:

i. G={ (S,A,B), (a,b), (S,P)} where P is the set of productions {S$\rightarrow$AB, S$\rightarrow$AA,

$A \to aB$, $A \to ab$, $B \to b\}$.

ii. G = {(S,A), (a,b,c), S, P} where P consists of the production $\{S \to aSb$, $Sb \to bA$, $abA \to C\}$

b. Draw the state diagram of a finite state machine M with the following state-table.

f, g	0	1
s_0	s_2,y	s_1,z
s_1	s_2,x	s_3,y
s_2	s_2,y	s_1,z
s_3	s_3,z	s_0,x

150. a. Find a grammar that generates the set of words $\{a^n b^n c^n \mid n \geq 1\}$.
b. Construct a Turing machine with tape symbols 0,1 and B that will replace all 0's in the bit string with 1's and will not change any of the 1's in the bit string.

151. a. Test whether the following formula:
$Q \vee (P \wedge \sim Q) \vee (\sim P \wedge \sim Q)$ is a Tautology or contradiction without constructing the truth table.
b. Using rule CP, derive $P \to (Q \to S)$ from $P \to (Q \to R), Q \to (R \to S)$.

152. a. Obtain the principle conjunctive normal form of $(\sim P \to R) \wedge (Q \Leftrightarrow P)$.
b. Prove that $(\exists x)(P(x) \wedge Q(x)) \Rightarrow (\exists x)P(x) \wedge (\exists x)Q(x)$.

153. a. Show that $f: R \to R$ defined by $f(x) = 2x - 3$ is a bijection and find its inverse. Compute $f^{-1} \circ f$ and $f \circ f^{-1}$.
b. If $f: A \to B$, $g: B \to C$ and $h: C \to D$ are functions then prove that $h \circ (g \circ f) = (h \circ g) \circ f$.

154. Let X = {1, 2, 3, 4} and a mapping $f: X \to X$ be given by $f: \{(1,2),(2,3),(3,4),(4,1)\}$. Form the composite functions f^2, f^3, f^4.

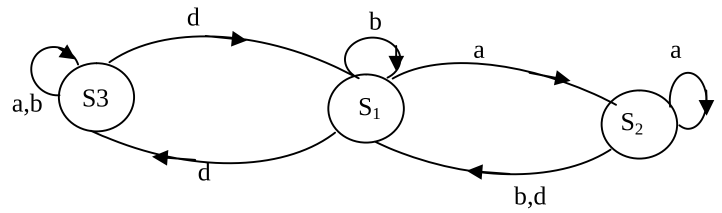
155. Define an equivalence relation and hence prove that the relation "congruence modulo m" over the set of +ve integers is an equivalence relation.

156. If R is the equivalence relation on the set A = {1, 2, 3, 4, 5, 6}, find the partition of A induced by R given below:
R = {(1, 1), (1, 2), (2, 1), (2, 2), (3, 3), (4, 4), (4, 5), (5, 4), (5, 5), (6, 6)}.

157. a. Prove that the cyclic group is abelian.
b. If $f = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \end{pmatrix}$ and $g = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 1 & 4 & 2 \end{pmatrix}$, find $f^{-1}gf$ and gfg^{-1}.

158. a. State and prove the necessary and sufficient condition for a nonempty subset H of a group (G,*) will be subgroup of G.
b. If G is the additive group of integers and H is a subgroup of G defined by $H = \{5x, x \in G\}$ find the distinct left cosets of H in G.

159. a. Let G = {V, T, P, S} where V = {*a, b, c, s*}, T = { *a, b, c*}, S is the start symbol and $P = \{s \to abs / bcs / bbs / a / cb\}$ derive the strings $bcbba, bbbcbba, bcabbbbbcb$.
b. Compute f_{bad}, f_{dad} and f_{badadd} from the finite state machine (S, I, *f*), S = {S_0, S_1, S_2}, I = {*a, b, d*} defined by the digraph.



160. Find the code words generated by the party check matrix when the coding function is $e: B^3 \to B^6$ $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

161. a. If A = { a, b, c, d, e } B = {a, b, e, g, h} C = {b, d, e, g, h, k, m, n
Verify $|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |B \cap C| - |C \cap A| + |A \cap B \cap C|$

b. Prove that $\sqrt{2}$ is irrational

162. a. Prove by mathematical induction for all n ≥ 1, 1+2+3+ . . . . + n = $\frac{n(n+1)}{2}$

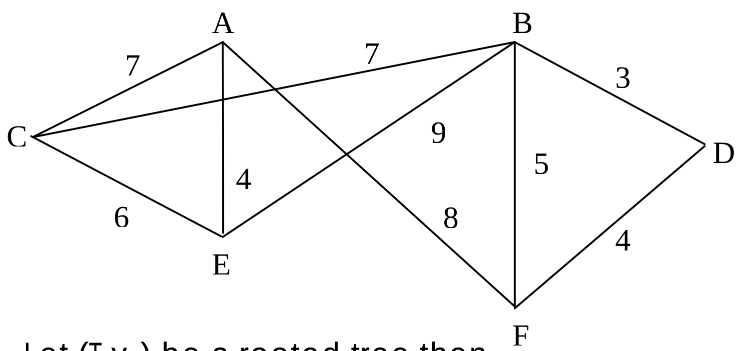
b. A box contains six red balls and four green balls. Four balls are selected at random from the box. What is the probability that two of the selected balls will be red and two will be green?

163. a. Let A = {1, 2, 3, 4} and let R = (1, 2), (2,3), (3,4), (2,1), find the transitive closure of R.

b. Let A = {1, 2, 3, 4} and B = {a, b, c} Let R = {(1,a), (1,b), (2,b), (2,c), (3,b), (4,a)} Let S = {(1,b), (2,c), (3,b), (4,b)} Compute $\overline{R}$, R∩S, R∪S and R^{-1}.

164. Let A = a_1, a_2, a_3 . . . . a_n be a finite set with n-elements n ≥ 2, then there are $\frac{n!}{2}$ even permutations and $\frac{n!}{2}$ odd permutations.

165. Find a minimum spanning tree of the labeled connected graph shown.



166. Let (T v_0) be a rooted tree then

a. There are no cycles in T

b. V_0 is the only root of T

c. Each vertex T, other than V_0 has in degree one and V_0 has in degree zero.

167. a. Let G be the set of all non zero real number and let $a * b = \frac{ab}{2}$. Show that (G *) is an abelian group.

b. In a complete graph with n vertices, there are $\frac{n-1}{2}$ edges disjoint Hamiltonian's Circuits, if n is an odd number ≥ 3.

168. a. Let the number of edges of G be m. Then G has is Hamittonian Circuit if $m \geq \frac{1}{2}(n^2 - 3m + 6)$. Where n is the number of vertices.

b. If a graph G has more than two vertices of odd degree then there can be no Eulers path in G.

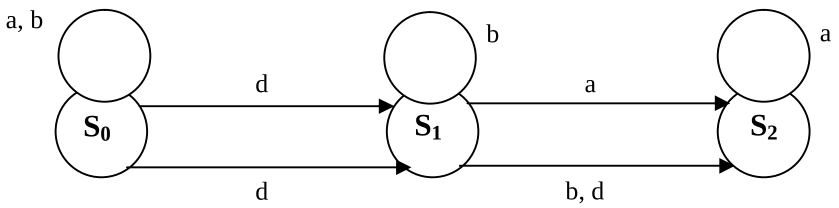
169. a. Consider (3, 8) encoding function e: $B^3 \rightarrow B^8$ defined by

e(0,0,0) = 00000000
e(0,0,1) = 10111000
e(0,1,0) = 00101101
e(0,1,1) = 10010101
e(1,0,0) = 10100100
e(1,0,1) = 10001001
e(1,1,0) = 00011100
E(1,1, 1) = 00110001

How many errors will 'e' detect?

b. Give $H = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ determine the group code $e_H = B^2 \rightarrow B^5$.

170. Let S = (s_0, s_1. s_2) I = (a, b, d) Consider the finite state machine M = (S I F) defined by the digraph shown in figure. Compute the function f_{bad}, f_{add} and f_{badadd} and verify that f_{add} = f_{bad} = f_{badadd}.



171. a. Show that $(\neg P \wedge (\neg Q \wedge R)) \vee (Q \wedge R) \vee (P \wedge R) \Leftrightarrow R$.

b. A survey shows that 57% of Indians like coffee whereas 75% like tea. Find the percentage of Indians who like both coffee and tea?

172. a. Given the relation matrices M_R and M_S. Find M_{RoS}, $M_{\overline{R}}$, $M_{\overline{S}}$, $M_{\overline{RoS}}$ and

show that $M_{\overline{RoS}} = M_{\overline{SoR}}$; $M_R = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$ $M_S = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix}$

b. Show that $((P \vee Q) \vee \neg(\neg P \wedge (\neg Q \vee \neg R))) \vee (\neg P \wedge \neg Q) \vee (\neg P \wedge \neg R)$ is a tautology.

173. a. Let X = {1, 2, 3, ..., 7} and let R = {(x, y) / x – y is divisible by 3} be a relation on X. Show that R is an equivalence relation.
b. If A = {α, β} and B = {1, 2, 3}. Find $(A \times B) \cap (B \times A)$.

174. Let A be a set of non zero integers and let R be a relation on A defined by (a, b) R (c, d) whenever ad = bc. Prove that R is an equivalence relation.

175. a. List all possible functions from X = {a, b, c} to Y = {0, 1} and indicate in each case whether the function is 1–1, onto or both.
b. Let (L, $\leq$) be a lattice in which $*$ and $\oplus$ denote the operations of meet and join respectively. For any a, b $\in$ L, $a \leq b \Leftrightarrow a * b = a \Leftrightarrow a \oplus b = b$.

176. a. Obtain the values of the Boolean forms $x_1 * (x_1^1 \oplus x_2)$, $x_1 * x_2$ and $x_1 \oplus (x_1 * x_2)$ over the ordered pairs of the two-element Boolean algebra.
b. Prove that every chain is a distributive lattice.

177. a. Prove that the set of all integers Z, define $*$ by $a * b = a + b + 1$ satisfies all the properties of a group.
b. Let S be the relation "divides" on B where B = {2, 3, 4, 6, 12, 36, 48}. Draw the Hasse diagram for the relation S.

178. a. Define Eulerian graph, Hamiltonian graph, weighted graph, spanning tree, minimal spanning tree, matching and colouring graph.
b. Prove that a group G is abelian iff $(a * b)^2 = a^2 * b^2$.

179. Construct a phase structure grammar G such that language L(G) of G is equal to language L where L = $\{ a^n b^n / n \geq 3 \}$.

180. a. Let $M = (S, I, F)$, where $S = \{s_0, s_1, s_2\}$, $I = \{0, 1\}$, and F is given by the following state transition table

	0	1
s_0	s_0	s_1
s_1	s_2	s_2
s_2	s_1	s_0

Find the state transition function corresponding to $w = 011$

b) Determine the group code $e_H : B^2 \to B^5$ where $H = \begin{bmatrix} 0 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

181. a. Prove that $\overline{A \cup B} = \overline{A} \cap \overline{B}$, where A and B are finite sets.
b. Use Euclidean Algorithm to compute GCD of (4389, 7293) and write it as sa +tb.

182. a. Prove that $((p \Rightarrow q) \wedge (q \Rightarrow r)) \Rightarrow (p \Rightarrow r)$ is a tautology.

b. Prove by mathematical induction $1^2 + 2^2 + 3^2 + \ldots\ldots + n^2 = \frac{n(n+1)(2n+1)}{6}$

183. a. Let s = { 1, 2, 3, 4} and A = S × S. Define the following relation R on A; (a,b) R (a′, b′) if and only if a+b = a′+b′

i. Show that R is an equivalence relation.

ii. Compute A/R.

b. Let $A=B=C=R$ (set of real numbers), let f:A→B, g:B→C be defined by f(a) = a –1 and g(b) = b^2, Find

i. fog (-2) ii. (gof)(2) iii. (fof)(y)

iv. (gog)(y) v. (fog)(x)

184. a. Let A = {1,2,3,4} and let R and S be the relations on A described by

$$M_R = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \text{and } M_S = \begin{bmatrix} 1 & 1 & 0 & 01 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$

Use Warshall's algorithm to compute the transitive closure of RVS.

b. Let A= {1, 2, 3, 4, 5, 6} and $P = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 4 & 3 & 1 & 5 & 6 \end{pmatrix}$

i. Write P as a product of disjoint cycles.

ii. Compute P^{-1}

iii. Compute P^2

iv. Find the smallest positive integer k, such that $P^k = I_A$.

185. a. Let D_{63} be the lattice of all positive divisors of 63 with divisibility relation.

i. Draw the Hasse diagram of the lattice.

ii. Prove or disprove the statement: D_{63} is a Boolean Algebra

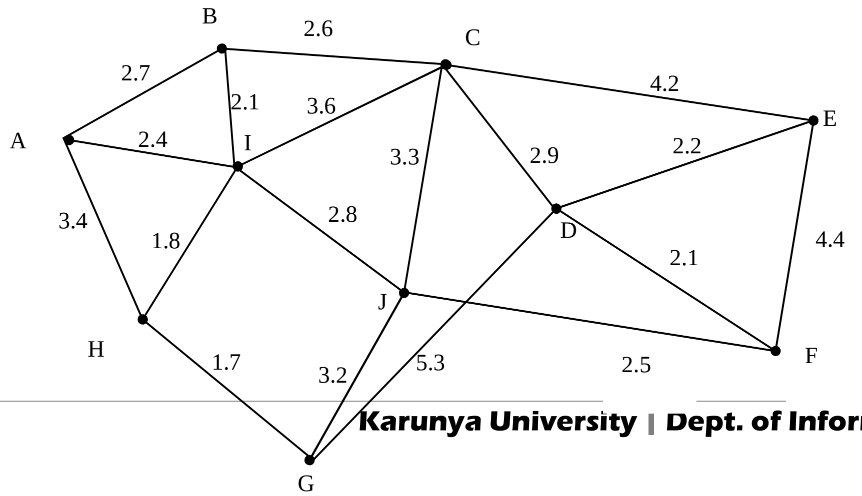
b. i. Define complemented lattice.

ii. Find complement of each element in D_{105}

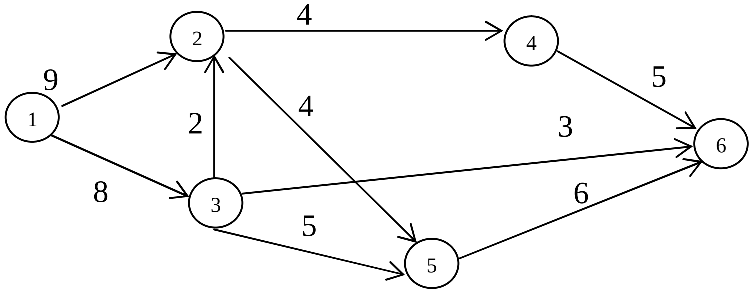
iii. Define distributive lattice, with example.

186. a. Consider the Boolean polynomial $p(x, y, z) = (x \wedge y') \vee (y \wedge (x' \vee y))$. If B = {0, 1}, compute the truth table of the function f: B_3→B defined by P.

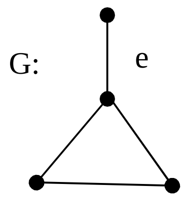
b. A communication company is investigating the costs of upgrading links between the relay stations if owns. The weighted graph given below, shows the stations and the cost in millions of dollars for upgrading each link. Use kruskal's algorithm and find a minimal spanning tree.



187. Find a maximum flow in the given network by using the labeling algorithm.



188. a. Find the chromatic polynomial of the graph given below using the edge 'e' and hence find the chromatic number of G.



b. Determine whether the set of integers together with the binary operation, a*b= a+b – ab is a semigroup that has an identity element.

189. a. Construct a phase structure grammar G, such that the language L(G) of G is equal to the language L. $L=\{a^n b^n / n \geq 1\}$

b. Let $H = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ be a parity check matrix. Determine (3,6) group code $e_H: B^3 \rightarrow B^6$.

190. a. Construct a Moore machine M that will accept exactly the string 001 from the input strings of 0′ s and 1′s

b. Consider the (3,5) group encoding function $e:B^3 \rightarrow B^5$ defined by

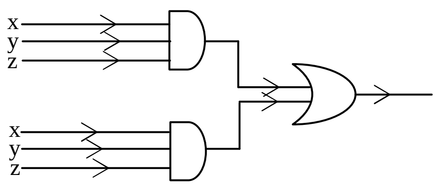
e(000) = 00000 e(100) = 10011
e(001) = 00110 e(101) = 10101
e(010) = 01001 e(110) = 11010
e(011) = 01111 e(111) = 11100

Decode the following words relative to a maximum likelihood decoding function.

i. 11001 ii. 01010 iii. 00100

191. a. Construct Truth table for $(P \leftrightarrow Q) \leftrightarrow (R \leftrightarrow S)$.

b. Find the output of the network given below and design a simpler network having the same output.



192. a. Show that if n is a positive integer, then prove that $1^2+2^2+3^2+\ldots\ldots = n(n+1)(2n+1)/6$.

b. Let X= {1,2,3,4,5,6,7} and R={(x,y)/(x-y) is divisible by 3}. Show that R is an equivalence relation and draw the graph of R.

193. a. Let R={(1,2),(2,3),(3,3),(3,4),(4,2)} and A={1,2,3,4}. Find the transitive closure of R.

b. Draw the Hasse diagram for the partial ordering {(a, b) such that a divides b} on {1,2,3,4,6,8,12,24}.

194. a. Prove that a simple graph with n vertices must be connected if and only if it has more than (n-1)(n-2)/2 edges.

b. Give example of a graph which is (i) Eulerian but not Hamiltonian (ii) Hamiltonian but not Eulerian (iii) Both Eulerian and Hamiltonian (iv) non Eulerian and non Hamiltonian.

195. a. If a simple graph G has "v" vertices and "e" edges, how many edges does G^c have?

b. Let G be a simple graph with "n" vertices. Show that if $\delta(G) \geq \frac{n}{2}$ then G is connected where $\delta(G)$ is minimum degree of the graph G.

196. a. Prove that G= (v, e, w) be a weighted connected undirected graph and v be partioned into two sets L and R. If e^* is a bridge of least weight between L and R, then there exists a minimal spanning tree for G that includes e^*.

b. Define Tree. Prove that a tree of a connected graph has no circuit.

197. a. Prove that if a linear graph has exactly one path between any two vertices, the linear graph is a tree and conversely.

b. Show that a tree with "n" vertices has (n-1) edges.

198. a. Construct a context free grammars for the language $L = \{a^n b^n / n \geq 1\}$.

b. Find the language generated by the context free grammar G= (N, T, S, P) where N= {s}, T = {a} and P={s→as/a}.

199. a. Design a Turing Machine to copy strings.

b. The state Table of a finite state machine M is given below

f, g	a	b
s_0	s_0, b	s_4, b
s_1	s_0, a	s_3, b
s_2	s_0, a	s_2, a
s_3	s_1, b	s_1, b
s_4	s_1, b	s_0, a

i) Find the input set I, the state set S, the output set O put the initial state of M.

ii) Draw the state diagram of M.

200. a. Show that the following premises are inconsistent
 i. If Jack misses many classes through illness then he fails high school.
 ii. If Jack fails high school, then he is uneducated.
 iii. If Jack reads a lot of books then he is not uneducated.
 iv. Jack misses many classes through illness and reads a lot of books.
 b. Obtain disjunctive normal form of $P \rightarrow \{(P \rightarrow Q) \wedge 7(7Q \vee 7P)\}$

201. a. Using mathematical induction prove that $a^n - b^n$ is divisible by $(a - b)$.
 b. Prove that the relation "congruence modulo m" over the set of positive integer is an equivalence relation.

203. a. Solve $a_n = 6a_{n-1} - 9a_{n-2}$ with $a_0 = 1$ and $a_1 = 6$ using recurrence relation.
 b. Let X = { 1,2,3,4,5,6,7} and R = {(x,y) / x – y is divisible by 3}. Show that R is an equivalence relation. Draw the graph of R.

204. a. Prove that the maximum number of edges in a simple graph with n vertices is $\frac{n(n-1)}{2}$
 b. How many edges are there in a graph with 10 vertices each of degree six?

205. Define Euler graph, Eulerian path, Hamiltonian graph and Hamiltonian path. Give an example of a graph which is
 a. Eulerian but not Hamiltonian
 b. Hamiltonian but not Eulerian
 c. Both Eulerian and Hamiltonian
 d. Non Eulerian and non Hamiltonian.

206. Draw the expression trees for the following expressions
 i. a(b+c) ii. (ab) + c iii. ab + ac iv. bb – 4ac
 v. $((a_3x + a_2)x + a_1)x + a_0$

207. a. Define Tree, Spanning tree and Binary tree.
 b. Draw a binary tree with seven vertices and only one leaf.
 c. Draw a binary tree with seven vertices and as many as leaves as possible.

208. a. Find the language L(G) over {a,b} generated by the grammar with productions $\sigma \rightarrow b\sigma$, $\sigma \rightarrow aA$, $A \rightarrow bA$, $A \rightarrow b$ where σ is the starting symbol.
 b. Design a finite state automaton that accepts precisely those strings over {a,b} that contains no a's.

209. Construct a finite state automaton that accepts those strings over{0,1} for which the last two input symbols are 1.

210. a. Without using truth table, prove that $(\neg P \vee q) \wedge (p \wedge (p \wedge q)) \equiv p \wedge q$.
 b. Find the number of integers between 1 and 250 both inclusive that are not divisible by any of the integers 2, 3, 5 and 7.

211. a. A die is tossed and the number showing on the top face is recorded. Let E, F and G be the following events. E: the number is at least 3, F: the number is at most 3, G: the number is divisible by 2.
 i) Are E and F mutually exclusive? Justify your answer.
 ii) Are G and F mutually exclusive? Justify your answer.
 iii) Is $E \cup F$ the certain event? Justify your answer.
 iv) Is $E \cap F$ the impossible event?.Justify your answer.
 b. Show by mathematical induction, that for all $n \geq 1$, $1+2+3+\ldots+n = \frac{n(n+1)}{2}$

212. a. Let A = {a, b, c, d, e} and R = {(a, a), (a, b), (b, c), (c, e), (c, d), (d, e)}, compute (i) R^2 (ii)R^∞.(7)
b. Let A = {1, 2, 3,4} and let R = { (1, 2) , (2 ,3), (3 ,4), (2, 1)}.Find the transitive Closure of R.

213. a. Let $P_1 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 7 & 3 & 2 & 1 & 4 & 5 & 6 \end{pmatrix}$ and $P_2 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 6 & 3 & 2 & 1 & 5 & 4 & 7 \end{pmatrix}$
i) Compute $P_1 \circ P_2$ (ii) Compute P_1^{-1} (iii) Is P_1 an even or odd permutation? Explain.
b. Let A = {1, 2, 3, 4} and B = {a, b, c} .Let R = {(1, a), (1, b), (2, b), (2, c), (3, b), (4, a)} And S = {(1, b) , (2 ,c), (3 ,b), (4, b) }.Compute (i) $\overline{R}$ (ii) $R \cap S$ (iii) $R \cup S$ (iv)R^{-1}.

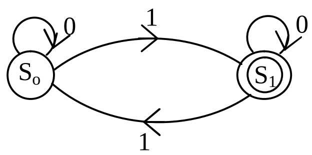
214. a. Draw the Hasse diagram of the lattice {P(S),$\subseteq$ } in which the join and meet are the operations $\cup$ and $\cap$ respectively, where S = {a, b, c}. Identify a sub lattice of this lattice with 4 elements and a subset of this lattice with 4 elements which is not a sub lattice.
b. In any Boolean algebra, show that a b′ + a′b = 0 if and only if a = b.

215. a. Prove that De Morgan's laws hold good for a complemented distributive lattice {L,$\vee$,$\wedge$}.
b. Simplify the following Boolean expressions using Boolean algebra.
(i)(x +y + xy) (x + z) (ii)x (y + z(xy +xz)′).

216. a. Prove that the number of edges in a bipartite graph with n vertices is at most $\frac{n^2}{2}$.
b. Show that the set Q^+ of all positive rational numbers forms an abelian group under the operation * defined by $a*b = \frac{ab}{2}$, a, b $\in Q^+$.

217. a. If S = N X N, the set of ordered pairs of positive integers with the operation * defined by (a, b)*(c, d) = (ad + bc, bd) and if f: (S,*) → (Q, +) is defined by $f(a, b) = \frac{a}{b}$, show that f is a semi group homomorphism.
b. Verify the handshaking theorem for the complete graph with n vertices .Verify also that the number of odd vertices in the graph is even.

218. Consider the Moore machine whose digraph is shown below. Describe in words the language L(M).Construct the regular expression that corresponds to L(M) and describe the production of the corresponding grammar G in BNF form.



219. a. Draw the state diagram for the Finite state automaton for which the state table is given below, the accepting states are S_1 and S_3.Find also whether the string aaababbab is accepted by this FSA.

I / S	a	b
S_0	S_1	S_2
S_1	S_2	S_1
S_2	S_2	S_3
S_3	S_1	S_0

b. Find the code words generated by the encoding function e: $B^2 \to B^5$ with respect to the parity check matrix H = $\begin{pmatrix} 0 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

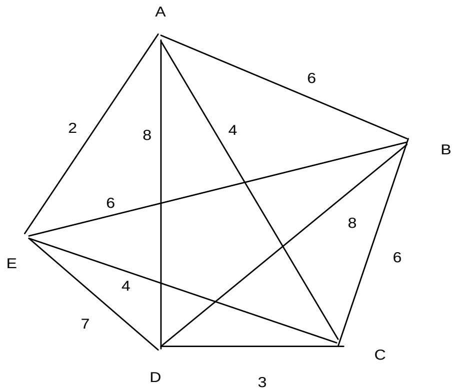
220. a. Let n be an integer. Prove that if n^2 is odd, then n is odd.
b. Prove by Mathematical Induction $a + ar + ar^2 + ... + ar^{n-1} = \frac{a(1-r^n)}{1-r}, \ for\ r \neq 1.$

221. a. Let A = $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ $and\ B = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$
(i) Compute $A \vee B$
(ii) Compute $A \wedge B$
(iii) Compute A ⊙ B^T.
b. Find GCD(190,34) using Euclidean Algorithm and write GCD = Sa+Tb.

222. a. Use Warshall's algorithm to find the transitive closure of R={(1,1),(1,4),(2,1),(2,2),(3,3),(4,4)} on the set A={1,2,3,4}.
b. Let f:A→B and g:B→A be functions such that g∘f=I_A and f∘g=I_B. Show that f is a one-to-one correspondence between A and B and g is a one-to-one correspondence between B and A and each is the inverse of the other.

223. a. Show that the Relation on the set of integers is such that aRb if and only if a ≡ b(mod m), where m is any positive integer >1 is an equivalence relation.
b. Let A = {1,2,3,4,5,6,7,8} and p = $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 3 & 4 & 6 & 5 & 2 & 1 & 8 & 7 \end{pmatrix}$ be a permutation of A.
i) Compute p^2 ii) Write p as a product of disjoint cycles
iii) Write p as a product of transpositions.

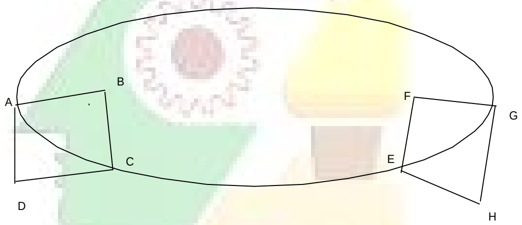
224. a. Find the minimal spanning tree for the weighted graph given below.



b. Show that if n is a positive integer and p^2/n, where p is a prime number, then D_n is not a Boolean algebra.

225. a. Construct the tree of algebraic expression (a-b) x (c + (d÷e)). Also find the preorder to this tree.
b. Consider the Boolean polynomial $p(x,y,z)= (x \wedge y) \vee (y \wedge z')$. If B={0,1}, compute the truth table of the function f: $B_3 \rightarrow B$ defined by p.

226. a. Use Fleury's algorithm to construct an Euler circuit for the graph given below.



b. Let G be a group of real numbers under addition and let G′ be the group of positive real numbers under multiplication. Let f: G→G′ be defined by $f(x)=e^x$. Show that f is an isomorphism. (7)

227. a. Show that (Z_4 ,$+_4$) is a semi group, where $+_4$ is defined as [a] $+_4$ [b]= [(a+b)mod4] .
b. Let M_R be the matrix of a suitability relation between 5men and 5 women. Use Hall's marriage theorem to find whether each man can marry a suitable women? $M_R = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix}$

228. a. Consider the (3,6) encoding function e: $B^3 \rightarrow B^6$ defined by
e(000)=000000
e(001)=001100
e(010)=010011

e(011)=011111
e(100)=100101
e(101)=101001
e(110)=110110
e(111)=111010
Show that this encoding function is a group code.

b. Let e: $B^m \rightarrow B^n$ be a group code. Show that the minimum distance of e is the minimum weight of a nonzero code word.

229. Consider the machine whose state transition table is

	a	b
S_0	S_0	S_4
S_1	S_1	S_0
S_2	S_2	S_4
S_3	S_5	S_2
S_4	S_4	S_3
S_5	S_3	S_2

and S= { $S_0, S_1, S_2, S_3, S_4, S_5$}. Show that R= {($S_0$, S_0),(S_0, S_2), (S_1, S_1),(S_1, S_3), (S_1, S_5), (S_2,S_0), (S_2,S_2), (S_3,S_1), (S_3, S_3),(S_3, S_5) ,(S_4, S_4),(S_5, S_1), (S_5, S_3),(S_5, S_5)} is a machine congruence. Construct the state transition table for the corresponding quotient machine.