

6. a) Solve the following finite difference equation.

$$Y_{n+2} - Y_{n+1} - 6Y_n = 0.$$

The given boundary conditions are $Y_0 = 0$ and $Y_1 = 1$.

Find the value of $\Delta^2 Y_4$.

- b) Two concentric spheres of radii R_1 and R_2 are separated by a solid material. The inner surface of the sphere whose radius is R_1 is maintained at a constant temperature T_1 and the outer surface of the system whose radius is R_2 is maintained at temperature T_2 . Deduce an expression for one-dimensional, steady-state temperature distribution within the separating material.

BACHELOR OF ENGINEERING IN CHEMICAL ENGINEERING
FINAL EXAMINATION, 2014

(4th Year, 1st Semester)

MATHEMATICAL MODELLING IN CHEMICAL ENGINEERING

Time : Three hours

Full Marks : 100

Answer any *five* questions.

All questions carry equal marks.

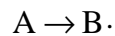
1. Consider a flat plate of length L and constant thermal conductivity k , in which heat is generated at the constant rate of GW/m^3 . The boundary surface at $x = 0$ is kept insulated and the boundary surface at $x=L$ is kept at zero temperature. Deduce the necessary differential equation for one dimensional, steady-state conductive heat transfer and solve it to find an expression for temperature distribution in the plate.
2. a) A tracer is used to characterise the degree of mixing in a continuous stirred tank. Water enters and leaves the mixer at a rate of $Q \text{ m}^3/\text{minute}$. The effective volume of water in the tank is $V \text{ m}^3$. At a time $t=0$, a mass $m_0 \text{ kg}$ of the tracer is injected into the tank, and the tracer concentration $C \text{ kg/m}^3$ in the outlet stream is monitored.

[Turn over

[2]

The injection is rapid enough so that all of the tracer may be considered to be in the tank at $t=0$. Deduce an expression to find C in terms of m_0 , V , Q and t .

- b) The water level in a municipal reservoir has been decreasing steadily during a dry spell. The rate of flow out is at steady value of 10^7 litres/day. The water input rate is $10^6 \cdot \exp(-t/100)$ litres/day where t is the time in days from the beginning of the dry spell at which time the reservoir contained 10^9 litres of water. Find out the value of water volume at the end of 60 days.
3. In the phenomenon of simultaneous diffusion and chemical reaction, consider a cylindrical porous catalyst of length L , placed in a catalytic reactor where it is submerged in a gas stream containing the reactant A and product B . The reactant diffuses into the pore. Inside the catalyst, a first order chemical reaction occurs given as



At the pore entrance of a particular catalyst, it may be assumed that the concentration is C_{AS} moles of A per unit volume. Formulate the problem mathematically and find an expression to show how the concentration of the reactant A changes with distance inside the catalyst.

[3]

4. Determine an expression for one dimensional, time-dependent temperature distribution $T(x, t)$ in a slab, $0 \leq x \leq L$, by solving the given heat conduction equation.

$$\frac{\partial T}{\partial t} = \alpha \cdot \frac{\partial^2 T}{\partial x^2} \text{ in } 0 \leq x \leq L, t > 0,$$

subject to the boundary and initial conditions shown below.

$$T = 0 \text{ at } x = 0, t > 0.$$

$$T = 0 \text{ at } x = L, t > 0.$$

$$T = T_0 \text{ for } t = 0 \text{ in } 0 \leq x \leq L.$$

In the problem, α is a constant and T_0 is a constant temperature. The given equation is to be solved by the method of separation of variables.

5. Consider a pin-fin whose base is attached to a wall at surface temperature T_s . The fin is cooled along its surface by a fluid at temperature T_∞ . The fin is of uniform cross-sectional area A , its thermal conductivity being k .

Mathematically formulate the problem for heat transfer through the fin and derive an expression for the temperature distribution when the fin is sufficiently long.

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