6. a) Solve the following finite difference equation.

$$Y_{n+2} - Y_{n+1} - 6Y_n = 0.$$

The given boundary conditions are $Y_0 = 0$ and $Y_1 = 1$. Find the value of $\Delta^2 Y_4$.

b) Two concentric spheres of radii R_1 and R_2 are separated by a solid material. The inner surface of the sphere whose radius is R_1 is maintained at a constant temperature T_1 and the outer surface of the system whose radius is R_2 is maintained at temperature T_2. Deduce an expression for one-dimensional, steady-state temperature distribution within the separating material.

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Ex/Ch-E/T/412/B/5/2014

Bachelor of Engineering in Chemical Engineering
Final Examination, 2014

(4th Year, 1st Semester)

Mathematical Modelling in Chemical Engineering

Time : Three hours | Full Marks : 100

Answer any ***five*** questions.

All questions carry equal marks.

1. Consider a flat plate of length L and constant thermal conductivity k, in which heat is generated at the constant rate of GW/m^3. The boundary surface at x = 0 is kept insulated and the boundary surface at x=L is kept at zero temperature. Deduce the necessary differential equation for one dimensional, steady-state conductive heat transfer and solve it to find an expression for temperature distribution in the plate.

2. a) A tracer is used to characterise the degree of mixing in a continuous stirred tank. Water enters and leaves the mixer at a rate of Q m^3/minute. The effective volume of water in the tank is V m^3. At a time t=0, a mass m_o kg of the tracer is injected into the tank, and the tracer concentration C kg/m^3 in the outlet stream is monitored.

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The injection is rapid enough so that all of the tracer may be considered to be in the tank at t=0. Deduce an expression to find C in terms of m_0, V, Q and t.

b) The water level in a municipal reservoir has been decreasing steadily during a dry spell. The rate of flow out is at steady value of 10^7 litres/day. The water input rate is 10^6. exp (–t/100) litres/day where t is the time in days from the beginning of the dry spell at which time the reservoir contained 10^9 litres of water. Find out the value of water volume at the end of 60 days.

3. In the phenomenon of simultaneous diffusion and chemical reaction, consider a cylindrical porous catalyst of length L, placed in a catalytic reactor where it is submerged in a gas stream containing the reactant A and product B. The reactant diffuses into the pore. Inside the catalyst, a first order chemical reaction occurs given as

$$A \rightarrow B \cdot$$

At the pore entrance of a particular catalyst, it may be assumed that the concentration is C_{AS} moles of A per unit volume. Formulate the problem mathematically and find an expression to show how the concentration of the reactant A changes with distance inside the catalyst.

4. Determine an expression for one dimensional, time-dependent temperature distribution T(x, t) in a slab, $0 \le x \le L$, by solving the given heat conduction equation.

$$\frac{\partial T}{\partial t} = \alpha . \frac{\partial^2 T}{\partial x^2} \text{ in } 0 \le x \le L, \ t > 0,$$

subject to the boundary and initial conditions shown below.

$T = 0$ at $x = 0$, $t > 0$.

$T = 0$ at $x = L$, $t > 0$.

$T = T_0$ for $t = 0$ in $0 \le x \le L$.

In the problem, α is a constant and T_0 is a constant temperature. The given equation is to be solved by the method of separation of variables.

5. Consider a pin-fin whose base is attached to a wall at surface temperature T_S. The fin is cooled along its surface by a fluid at temperature T_∞. The fin is of uniform cross-sectional area A, its thermal conductivity being k.

Mathematically formulate the problem for heat transfer through the fin and derive an expression for the temperature distribution when the fin is sufficiently long.