**11**

DE–3552

DISTANCE EDUCATION

M.Phil. DEGREE EXAMINATION, MAY 2008.

Mathematics

COMMUTATIVE ALGEBRA

(Upto 2006 batch)

Time : Three hours Maximum: 100 marks

Answer any FIVE questions.  
 (5 × 20 = 100)

1. (a) Define nil radical. Prove that the nil radical  of  is the intersection of all the prime ideal of .
   1. (b) Explain Tensor and Flat modulus.
2. (a) Define primary. Let  be a *p* - primary ideal and . Then
   1. (i) If  then ;
   2. (ii) If  then  is *p*-primary and .
   3. (iii) If  then .
   4. (b) State and prove that the uniqueness theorem.
3. (a) If  are -modulus then 
   1. (b) If  are submodulus of  then
   2. .
4. Let  be an exact sequence of -modules. Then
   1. (a)  is Noetherian   and  are Noetherian.
   2. (b)  is Artinian  and  are Artinian.
5. (a) Define Length.
   1. Let  be a ring in which the zero ideal is a product  of (not necessarily district) maximal ideals. Then  is Noetherian iff  is Artinian.
   2. (b) State and prove Jordan Holder Theorem.
6. (a) State and prove Going-up Theorem.
   1. (b) Let  be a maximal element of . Then  is a valuation ring of the field .
7. State and prove Norther’s normalization theorem.
8. (a) Define Discrete valuation ring.
   1. (b) Let  be a Noetherian local domain of dimension   
      1, *m* its maximal ideal and  its residue field. Then the following are equivalent.
   2. (i)  is a discrete valuation ring.
   3. (ii)  is integrally closed.
   4. (iii)  is a principal ideal.
   5. (iv) 
   6. (v) Every non zero ideal is a power of .
   7. (vi) There exists  such that every non zero ideal is of the form .

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DE–3553

**12**

DISTANCE EDUCATION

M.Phil. (Mathematics) DEGREE EXAMINATION, MAY 2008.

MEASURE THEORY

(upto 2006 batch)

Time : Three hours Maximum : 100 marks

Answer any FIVE questions.

Each question carries 20 marks.

1. (a) State and prove Lebesque monotone convergence theorem.
   1. (b) (i) If  and , *S* is closed in the complex plane and  lies in *S* for each  with  then  for almost all .
   2. (ii) If  are measurable for  and  then almost all lie in atmost finitely many 's.
2. Explain briefly integration of complex functions.
3. If  is a sequence of complex measurable functions on *X* such that  pointwise and  where  then  and  as  and so .
4. State and prove Jensen's inequality.
5. (a) Define conjugate exponents.
   1. (b) State and prove Schwarz's inequalities.
6. (a) If  are conjugate components  and if  and  then  and .
   1. (b) Suppose  and ,  then  and .
7. Let *S* be the class of all complete measurable simple functions on *X* such that  then *S* is a dense in .
8. (a) State and prove Radon-Niscodyin theorem.
   1. (b) Explain product measures.

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DE–3554

**13**

DISTANCE EDUCATION

M.Phil. DEGREE EXAMINATION, MAY 2008.

Mathematics

TOPOLOGICAL VECTOR SPACES

(Upto 2006 Batch)

Time : Three hours Maximum : 100 marks

Answer any FIVE questions.

Each question carries 20 marks.

1. (a) Let  be a topological vector space. Prove the following
   1. (i) If  then , where  runs through all neighborhoods of .
   2. (ii) If  and  then .
   3. (iii) If  is a subspace of , so is .
   4. (b) Prove that every locally compact topological vector space  has finite dimension. (10 + 10)
2. (a) In a topological vector space , prove the following
   1. (i) every neighborhood of  contains a balanced neighborhood of  and
   2. (ii) every convex neighborhood of  contains a balance convex neighborhood of .
   3. (b) Suppose  is a subspace of a topological vector space  and  is locally compact, in the topology inherited from . Prove that  is a closed subspace of . (10 + 10)
3. (a) Suppose  is a convex absorbing set in a vector space . Prove the following
   1. (i) 
   2. (ii)  if 
   3. (iii)  is a seminorm if  is bounded
   4. (iv) If  and  then  and .
   5. (b) Prove that a topological vector space  is normable if and only if its origin has a convex bounded neighborhood.   
       (12 + 8)
4. (a) State and prove the category theorem.
   1. (b) State and prove the Banach–Steinhaus. (10 + 10)
5. (a) State and prove the closed graph theorem.
   1. (b) Suppose  is bilinear and separately continuous,  is an *F-*space and  and  are topological vector spaces. Prove that  in  when ever  in  and  in .
6. State and prove the Banach-Alaoglu theorem.
7. (a) Suppose  and  are normed space. Associate to each  the number . Prove that if  is a Banach space, so is .
   1. (b) If  is a separable topological vector space, if  and if  is weak–compact prove that  is metrizable, in the weak–topology. (10 + 10)
8. (a) If  is a locally convex space and  is the convex hull of a totally bounded set . Prove that  is totally bounded.
   1. (b) Let  be a compact group, suppose  and define  to be the convex hull of the set of all left translates of . Prove that
   2. (i) *f* is uniformly continuous and
   3. (ii)  is a totally bounded subset of . (8 + 12)
9. (a) Suppose  is the closed unit ball of a normed space . Define  for every . Prove the following :
   1. (i) This norm makes  into a Banach space.
   2. (ii) Let  be the closed unit ball of . For every  . Consequently  is a bounded linear functional on , of norm .
   3. (iii)  is weak–compact.
   4. (b) (i) If  is a continuous projection in a topological vector space prove that .
   5. (ii) Conversely, if  is an *F-*space and if  prove that the projection  with range  and null space  is continuous. (10 + 10)

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**14 A**

DE–3555

DISTANCE EDUCATION

M.Phil. (Mathematics) DEGREE EXAMINATION, MAY 2008.

FUNDAMENTALS OF DOMINATION IN GRAPH

(Upto 2006 batch)

Time : Three hours Maximum : 100 marks

Answer any FIVE questions

All questions carry equal marks.

* 1. (5 × 20 = 100)

1. For any graph *T* show that the following are equivalent
   1. (a) *T* is connected and acyclic
   2. (b) *T* is connected and 
   3. (c) *T* is acyclic and 
   4. (d) For any two vertices  and  in
   5.  there exists a unique  path.
2. (a) State and prove ORE theorem on dominating set.
   1. (b) Show that every connected graph  of order  has a dominating set  whose complement  is also a dominating set.
3. Show that if a connected graph  satisfies  then  for some  to 6.
4. Show that if a graph  has  then .
5. (a) Show that for any graph  and hereditary   
   property , 
   1. (b) Show that for any graph , .
6. Show that for any graph , .
7. Assume that the degrees  of vertices  of a graph satisfy .... . Show that if *t* is the largest integer for which  where  then .
8. For any graph , show that
   1. (a) If is connected then 
   2. (b) If  is connected then 
   3. (c) If  then .

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**14 B**

DE–3556

DISTANCE EDUCATION

M.Phil. (Mathematics) DEGREE EXAMINATION, MAY 2008.

DATA STRUCTURES AND ALGORITHMS

(Upto 2006 Batch)

Time : Three hours Maximum : 100 marks

Answer any FIVE questions.

All questions carry equal marks.

1. (a) Describe the concept of polymorphism with an example. (10)
   1. (b) Explain the following concepts with suitable example.
   2. (i) Dynamic binding
   3. (ii) Message passing. (5 + 5)
2. Explain the various statements available in C++ with suitable examples. (20)
3. (a) Explain how a multi-dimensional array can be implemented in a computer memory. (10)
   1. (b) (i) Explain the structure of singly linked list. (5)
   2. (ii) Write an algorithm to insert a node at the front of the linked list. (5)
4. (a) Explain doubly linked list concepts and its application. (10)
   1. (b) Write short notes on circularly linked list with example. (10)
5. (a) Write down the stack operations with example. Discuss the applications of stack. (10)
   1. (b) How are stacks used in evaluating a given numerical expression? Illustrate with example. (10)
6. (a) Write algorithms to do the following :
   1. (i) delete an element from the queue. (5)
   2. (ii) insert an element into the queue. (5)
   3. (b) Explain the following :
   4. (i) Priority queue (5)
   5. (ii) Circular queue. (5)
7. (a) Write an algorithm for converting general trees to binary trees. (10)
   1. (b) Write an algorithm for postorder tree traversal with example. (10)
8. (a) Explain the travelling salesman problem using BFS technique. (10)
   1. (b) Write and explain Kruskal’s algorithm to find a minimum cost spanning tree. (10)
9. ————————