AF-2760 **MMA1C1**

M.Sc. DEGREE EXAMINATION, NOVEMBER 2010

First Semester

Mathematics

ALGEBRA—I

(CBCS—2008 onwards)

Time : 3 Hours Maximum : 75 Marks

Part - A $(10 \times 2 = 20)$

Answer **all** the questions.

1. Suppose H and K are subgroups of a group G of order 10 and 21 respectively. Find o(HK).

2. Show that the intersection of two normal subgroups of a group G is also a normal subgroup of G.

3. Define the internal direct product of subgroups of a group and express $\mathbb{Z}/6\mathbb{Z}$ as internal direct product of its proper subgroups.

4. Find the class equation of S_3.

5. Show that if $a, b \in R$, then

(i) $a0 = 0a = 0$.

(ii) $a(-b) = -(a)b = -(ab)$ and

(iii) $(-a)(-b) = ab$.

6. Prove that any field is an integral domain.

7. If I is a ideal of a ring R containing the unit element, show that I = R.

8. Show that the intersection of two ideals is again an ideal.

9. Prove that an Euclidean ring possess an unit element.

10. Let R be an integral domain with unit element. Prove that if for $a, b \in R, a \mid b$ and $b \mid a$ then both a and b are associates in R.

Part - B (5 × 5 = 25)

Answer **all** questions.

11. (a) Prove that A_n is a normal subgroup of S_n with index 2.

(*Or*)

(b) If G is a finite group and $H \neq G$ is a subgroup of G such that o(G) does not divide i(H), then show that H contains a nontrivial normal subgroup of G.

12. (a) Let o(G) = p^{n}, where p is a prime number. Prove that G has a nontrivial center.

(*Or*)

(b) Show that any two p-sylow subgroups of a group G are conjugate.

13. (a) Show that a finite integral domain is a field.

(*Or*)

(b) Prove that a ring homomorphism $\phi : R \to R'$ is one to one if and only if the kernel of ϕ is zero submodule.

14. (a) Let I and J be ideals of a ring R. Show that IJ, the set of finite sum of elements of the form ab where $a \in \mathrm{I} b \in$ J is an ideal. Also show that $\mathrm{IJ} \subset \mathrm{I} \cap \mathrm{J}$.

(*Or*)

(b) Show that the set of multiples of a fixed prime number p form a maximal ideal of the ring of integers.

15. (a) If p is a prime number of the form $4n+1$, then prove that $p = a^2 + b^2$ for some integers a,b.

(*Or*)

(b) State and prove the division algorithm for polynomials.

Part - C $(3 \times 10 = 30)$

Answer any **three** questions.

16. State and prove the Cauchy's theorem for abelian groups.

17. Let G be a group and suppose that G is a internal direct product of $N_1, N_2 \ldots N_n$. Let $T = N_1 \times \ldots \times N_n$. Then prove that G and T are isomorphic.

18. Show that the ring of quaternion is a non-commutative division ring.

19. Show that a commutative ring with unit element R is a field if and only if the only ideals of R are (0) and R itself.

20. Show that if R is a unique factorization domain then R[x] is also an unique factorization domain.

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AF-2761 **MMA1C2**

M.Sc. DEGREE EXAMINATION, NOVEMBER 2010

First Semester

Mathematics

ANALYSIS—I

(CBCS—2008 onwards)

Time : 3 Hours Maximum : 75 Marks

Part - A (10 × 2 = 20)

Answer **all** questions.

1. Define limit point of a set and give an example of a set which has no limit point.

2. Give an example to show that disjoint sets need not be separated.

3. When do you say that a series converges ? Give an example of a divergent series.

4. Define upper and lower limits of a sequences.

5. Give an example of a power series with radius of convergence zero.

6. State the partial summation formula.

7. Give an example of a function which has second kind discontinuity at every point.

8. Monotomic function have no discontinuities of the second kind. Why ?

9. Define the derivation of a real function.

10. Define local maximum and local minimum of a real function defined on a metric space.

Part - B (5 × 5 = 25)

Answer **all** questions.

11 (a) Let A be the set of all sequences whose elements are the digits 0 and 1. Prove that A is uncountable.

(*Or*)

(b) Prove that compact subset of metric space are closed.

12. (a) Prove that if $\{p_n\}$ is a sequence in a compact metric space X, then some subsequence of $\{p_n\}$ converges to a point of X.

(*Or*)

(b) Prove that (i) $\lim_{n \to \infty} \sqrt[n]{p} = 1$ if $p > 0$ and

(ii) $\lim_{n \to \infty} \sqrt[n]{n} = 1$.

13 (a) If $\sum a_n = \text{A}$ and $\sum a_n = \text{B}$ prove that $\sum (a_n + b_n) = \text{A} + \text{B}$ and $\sum \text{C}a_n = \text{CA}$ for any fixed C.

(*Or*)

(b) If $\sum a_n$ is a series of complex numbers which converges absolutely, prove that every rearrangement of $\sum a_n$ converges, and they all converge to the same sum.

14 (a) Let X, Y, Z be metric spaces, $\text{E} \subset \text{X}$ f maps E into Y, g maps range of $f, f(\text{E})$ int z and h maps E into Z defined by $h(x) = g(f(x))$; $x \in \text{E}$. If f is continuous at a point $p \in \text{E}$ and if g is continuous at the point $f(p)$, prove that h is continuous at p.

(*Or*)

(b) Suppose f is a continuous 1 – 1 mapping of a compact metric space X onto a metric space Y. Prove that the inverse mapping f^{-1} defined on Y by $f^{-1}(f(x)) = x$; $x \in \text{X}$ is a continuous mapping of Y onto X.

15 (a) Let f be defined on [a, b] ; if f has a local maximum at a point $x \in (a, b)$ and if $f'(x)$, prove that $f'(x) = 0$.

(*Or*)

(b) State and prove the generalized mean value theorem.

Part - C (3 × 10 = 30)

Answer any **three** questions.

16. If a set E is R^k has one of the following three properties, then prove that it has the other two :

(a) E is closed and bounded

(b) E is compact

(c) Every infinite subset of E has a limit point in E.

17. State and prove (a) Root test ; (b) Ratio test.

18. Suppose :

(i) $\sum_{n=0}^{\infty} a_n$ converges absolutely

(ii) $\sum_{n=0}^{\infty} a_n = A$

(iii) $\sum_{n=0}^{\infty} b_n = B$

(iv) $C_n = \sum_{k=0}^{n} a_k b_{n-k} \quad (n = 0, 1, 2, \ldots)$

prove that $\sum_{n=0}^{\infty} C_n = AB$.

19. Let f be a continuous mapping of a compact metric space X into a metric space Y. Prove that f is uniformly continuous on X.

20. State and prove the L'Hospital's rule.

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AF-2762 | **MMA1C3**

M.Sc. DEGREE EXAMINATION, NOVEMBER 2010

First Semester

Mathematics

DIFFERENTIAL GEOMETRY

(CBCS—2008 onwards)

Time : 3 Hours | Maximum : 75 Marks

Part - A $(10 \times 2 = 20)$

Answer **all** questions.

1. What is meant by 'osculating plane' ? Explain

2. Define the binormal line at a point on a curve.

3. Define involutes and evolutes.

4. Define :

(i) Cylindrical helix ; and

(ii) Circular helix.

5. Define 'ordinary point'.

6. State the formulae to determine the angle between two parametric curves.

7. Prove that the curves of the family $\frac{v^3}{u^2}$ = constant are geodesics on a surface with metric $v^2\, du^2 - 2\, uv\, du\, dv + 2u^2\, dv^2$ $(u > 0, v > 0)$.

8. Explain the term 'the geodesic curvature vector' of a curve.

9. State Rodrigue's formula.

10. Define 'polar developable' and 'rectifying developable' of a space curve.

Part - B (5 × 5 = 25)

Answer **all** questions.

11. (a) Find the curvature and torsion of the curve $\vec{\gamma} = (3u - u^3, 3u^2, 3u + u^3)$.

Or

(b) Prove that a necessary and sufficient condition that a curve be plane is

$$\left[\dot{\vec{\gamma}}, \ddot{\vec{\gamma}}, \dddot{\vec{\gamma}}\right] = 0.$$

12. (a) Determine the intrinsic equation of the curve

$$\vec{\gamma} = (a\,e^{u}\cos u, a\,e^{u}\sin u, b\,e^{u})$$

Or

(b) Prove that the following is a characteristic property of helices : the ratio of the curvature to the torsion is constant at all points.

13. (a) Derive the equations of a right helicoid and the general helicoid in terms of the position vector of a general point.

Or

(b) Show that a metric is invariant under a parameter transformation.

14. (a) Prove that, on the general surface, a necessary and sufficient condition for the curve $v = c$ be a geodesics is $E\,E_2 + F\,E_1 - 2\,E\,F_1 = 0$ when $v = c$, for all values of u.

Or

(b) Derive the canonical equations for geodesics.

15. (a) Show that the edge of regression of the osculating developable is the curve itself.

Or

(b) Show that the edge of regression of the rectifying developable has the equation.

$$\vec{R} = \vec{r} + k\,\frac{(\tau\,\vec{t} + k\,\vec{l}}{k^1\,\tau - k\,\tau^1} \quad .$$

Part - C (3 × 10 = 30)

Answer any **three** questions.

16. Find the equation of the osculating plane at a point on the cubic curve given by $\vec{r} = (u, u^2, u^3)$ and prove that the osculating planes at any three points of the curve meet at a point buying in the plane determined by these three points.

17. State and prove fundamental existence theorem for space curves.

18. Prove that on the surface with metric

$ds^2 = u^2\, du^2 + u^2\, dv^2$ the family of curves orthogonal to UV = constant is given by $\frac{u}{v} = \text{constant}$. Also find the metric referred to new parameters so that these two families are parametric.

19. Prove that a characteristic property of a geodesic is that at every point its principal normal is normal to the surface and every curve having this property is a geodesic

20. Show that a necessary and sufficient condition for a surface to be a developable is that its Gaussian curvature shall be zero.

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AF-2763

MMA1C4

M.Sc. DEGREE EXAMINATION, NOVEMBER 2010

First Semester

Mathematics

DIFFERENTIAL EQUATIONS

(CBCS—2008 onwards)

Time : 3 Hours Maximum : 75 Marks

Part - A $(10 \times 2 = 20)$

Answer **all** questions.

1. Find two linearly independent solutions of $y'' + \dfrac{1}{4x^2}\; y = 0, \; (x > 0)$.

2. Find the values of $p_n\,(1)$ and $p_n\,(-1)$.

3. Compute the indicial polynomial and its roots of the equation $x^2\,y'' + xy' + (x^2 - \alpha^2)\,y = 0$, where α is a non-negative constant.

4. Find the singular point of the equation $x^2 y'' + axy' + by = 0$, where a, b are constants and verify whether it is a regular singular point.

5. Explain the difference between general integral and complete integral of a partial differential equation.

6. Write any two non-linear partial differential equations.

7. Find a particular integral of the equation

$$\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} = 2y - x^2.$$

8. State interior and exterior Dirichlet problems.

9. Obtain a PDE by eliminating arbitrary functions f and g from $u = f(x + iy) + g(x - iy)$.

10. Find the solution of the PDE $p\,q\,z = p^2 (xq + p^2) + q^2 (yp + q^2)$.

Part - B (5 × 5 = 25)

Answer **all** questions.

11. (a) Show that $\int_{-1}^{1} P_n(x)\, P_m(x)\, dx = 0$, when $n \neq m$.

Or

(b) Find the solution θ of $y'' + (x-1)^2 y' - (x-1)\, y = 0$ in the form $\phi(x) = \sum_{k=0}^{\infty} C_k (x-1)^k$, which satistics $\phi(1) = 1, \phi'(1) = 0$.

12. (a) Obtain two linearly independent solutions of the equation $x^2 y'' - 2x(x+1)y' + 2(x+1)y = 0$.

Or

(b) Show that between any two positive zeros of J_α there is a zero of $J_{\alpha+1}$.

13. (a) Show that the equations $xp - x = yq$ and $x^2 p - xz - q$ are compatible and find their solution.

Or

(b) Find the complete integral of the equation $(p^2 + q^2)y = az$.

14. (a) Solve the equation $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial x \partial y} - 6 \frac{\partial^2 z}{\partial y^2} = y \cos x.$

Or

(b) Reduce the equation $\frac{\partial^2 z}{\partial x^2} + 2 \frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = 0$ to canonical form and solve it.

15. (a) Show that in two-dimensional case it is possible to reduce Neumann problem to the Dirichlet problem.

Or

(b) A rigid sphere of radius 'a' is placed in a stream of fluid whose velocity in the undisturbed state inV. Determine the velocity of the fluid at any point of the disturbed stream.

Part - C (3 × 10 = 30)

Answer any **three** questions.

16. Show that there is basis ϕ_1, ϕ_2 for the solutions of $x^2 y'' + 4xy' + (2 + x^2) y = 0$, $(x > 0)$ of the form $\phi_1 (r) \frac{\psi_1 (r)}{x^2}$, $\phi_2 (r) = \frac{\psi_2 (r)}{x^2}$. Also find all solutions of $x^2 y'' + 4xy' + (2 + x^2) y = x^2$ for $x > 0$.

17. Show that the equation $xy'' + (1-x) y' + \alpha y = 0$. where α is a constant has a regular singular point at $x = 0$. Compute the indicial polynomial and its roots. Find a solution ϕ of the form $\phi (x) = x^r \sum_{k=0}^{\infty} C_k x^k$.

18. Find the integral surface of the linear partial differential equation

$$x\,(y^2 + z)\,p - y\,(x^2 + z)\,q = (x^2 - y^2)\,z$$ which contains the straight line $x + y = 0$, $z = 1$.

19. Show that in spherical polar co-ordinates r, θ, ϕ Laplace's equation possesses solutions of the form $\left\{ Ar^n + \frac{B}{r_{n+1}} \right\} \theta \cos(\theta)\, e^{\pm im\phi}$ where A, B, m and n are constants and $\theta\ (\mu)$ statistics the ordinary differential equation

$$(1-\mu^2)\frac{d^2\theta}{d\ \ ^2} - 2\quad \frac{d\theta}{d} + \left\{ n\,(n+1) - \frac{m^2}{1- \quad ^2} \right\} \theta = 0.$$

20. Obtain d' Alembert's solution of the one-dimensional wave equation $\frac{\partial^2 y}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 y}{\partial t^2}$.

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AF-2764 **MMA1E1**

M.Sc. DEGREE EXAMINATION, NOVEMBER 2010

First Semester

Mathematics

ELECTIVE MECHANICS

(CBCS—2008 onwards)

Time : 3 Hours Maximum : 75 Marks

Part - A (10 × 2 = 20)

Answer **all** Questions.

1. Show that for a single particle if the mass varies with time, the equation of motion implies $\frac{d}{dt}(m\mathrm{T}) = \overline{\mathrm{F}} \cdot \overline{p}$.

2. A sphere rolling down on a rough inclined plane without shipping. State the name of the constraint with reason.

3. Find the equation of motion of single particle using Cartesian co-ordinates.

4. What do you mean by cyclic coordinates ?

5. State Hamilton's principle.

6. Define configuration space and phase space.

7. State Kepler's law of planetary motion.

8. Show that the elliptic circuit is invariant under reflection about the aspidal vectors.

9. Write the equation of first integral of motion.

10. What is the direction and magnitude of the Laplace-Runge-Lenz vector.

Part - B (5 × 5 = 25)

Answer **all** Questions.

11 (a) Define angular momentum of a system of particle. State and prove the principle of conservation of angular momentum.

(*Or*)

(b) Describe the Atwood's machine and derive the equation of motion.

12. (a) Obtain Lagrange's equation of the Rayleigh dissipation function.

(*Or*)

(b) Obtain Lagrange's equation of motion of a head sliding on a uniformly rotating wire in a force free space.

13 (a) Show that the shortest distance between two points in a plane is a straight line.

(*Or*)

(b) Discuss the motion of a hoop rolling without slipping down on an inclined plane. Find the acceleration and frictional force of constraint.

14 (a) With usual notation, prove that

$$\frac{1}{2} m\dot{r}^2 + \frac{1}{2}\frac{l^2}{mr^2} + V = \text{constant.}$$

(*Or*)

(b) State and prove Virial theorem.

15 (a) Use Kepler's equation $wt = \psi - e\sin\psi$, prove that $\tan\frac{\theta}{2} = \sqrt{\frac{1+e}{1-e}}\ \tan\frac{\psi}{2}$

(*Or*)

(b) A particle describe a circular circuit under the influence of an attractive central force directed towards a point on the circle. Show that the force varies as the inverse fifth power of the distance.

Part - C (3 × 10 = 30)

Answer any **three** Questions.

16. State D' Alembert's principles and hence obtain the Lagrange's equation of motion for a holonomic system.

17. Derive the motion of a charged particle in an electromagnetic field in the form $v = q_x - \frac{q}{e}\left(\bar{\mathrm{A}} \cdot \bar{v}\right)$ with $\mathrm{F}_x = \frac{-\partial \mathrm{U}}{\partial x} + \frac{d}{dt}\left(\frac{\partial v}{\partial v_\theta}\right)$.

18. Derive Euler-Lagrange differential equations.

19. Discuss Kepler's problem. For the inverse square law of forces, prove that the circuit is a conic with $e = \left(1 + \frac{2\mathrm{E}l^2}{mk^2}\right)^{3/2}$ and find the nature of the circuit that depends on the magnitude of e and find the condition for circular motion.

20. For a circular and parabolic orbits in an attractive $\frac{1}{r}$ potential hairs the same angular momentum. Show that the perihelion distance of the parabola is one half of the radius of the circle.

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AF-2765	**MMA1E2**

M.Sc. DEGREE EXAMINATION, NOVEMBER 2010

First Semester

Mathematics

Elective—PROGRAMMING IN C++

(CBCS—2008 onwards)

Time : 3 Hours Maximum : 60 Marks

Part - A $(10 \times 1\frac{1}{2} = 15)$

Answer **all** questions.

1. Differentiate between the constructs ‘structure” and “class” in C++.

2. Define Inline function.

3. Why friend function is used ?

4. Define copy constructor.

5. List various types of Inheritance.

6. Mention the advantages of "Abstract class".

7. Differentiate "call by value" and "call by reference".

8. List down the various classes used to perform I/O functions using files in C++.

9. Write a C++ function to find the biggest number among given three values.

10. What is a container class ?

Part - B (5 × 3 = 15)

Answer **all** questions.

11 (a) Describe various looping statements used in C++.

(*Or*)

(b) Write a short program in C++, which reads an arbitrary integer and finds all of its prime factors.

12. (a) Define static data member. Give example.

(*Or*)

(b) Write a short function in C++ which calculates the matrix product, given two 3 × 3 float arrays representing matrices as arguments.

13 (a) Give an example for parameterized constructor and explain it.

(*Or*)

(b) Write a program to overload binary operator.

14 (a) What is a container class ? What are the types of container classes ?

(*Or*)

(b) What is a friend function ? Illustrate with an example.

15 (a) What is meant by multiple inheritance ? Explain with simple C++ program constructs.

(*Or*)

(b) Explain exception handling.

Part - C (3 × 10 = 30)

Answer any **three** questions.

16. Explain in detail about various characteristics of OOPs.

17. How will you transfer values from derived class constructor to base class constructor ? Explain with suitable examples.

18. Write a C++ program, to count the number of characters, blanks and lines in a given text file.

19. Write a program in C++ to multiply two matrices A [$m \times n$] and B [$n \times p$]

20. Explain with example how exception handling are implemented using C++. In that explain how terminate ( ) and unexpected ( ) functions are used.

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AF-2766	MMA1E3

M.Sc. DEGREE EXAMINATION, NOVEMBER 2010

First Semester

Mathematics

Elective—CALCULUS OF VARIATIONS AND SPECIAL FUNCTIONS

(CBCS—2008 onwards)

Time : 3 Hours Maximum : 75 Marks

Part - A (10 × 2 = 20)

Answer **all** questions.

1. State the conditions for the strict maximum and strict minimum of a functional.

2. Find the extremals of the functional

$$v\left[y(x)\right] = \int_{0}^{\pi/2} \left[(y^{1})^{2} - y^{2}\right] dx \; ; \; y(0) = 0, \; y\left(\frac{\pi}{2}\right) = 1.$$

3. Find the Ostrogradsky equation for the functional

$$v\left[z\,(x,\,y)\right]=\iint_{\mathrm{D}}\left[\left(\frac{\partial z}{\partial n}\right)^2+\left(\frac{\partial z}{\partial y}\right)^2\right]dx\,dy.$$

4. Give an example of a variational problem in parametric form.

5. Define the terms :

   (a) Generalized co-ordinates ;

   (b) Generalized velocities ; and

   (c) Lagrangian function.

6. Explain the underlying idea of Ritz method.

7. Write the expression for $J_n(x)$ and show that $J_{-n}(x) = (-1)^n J_n(x)$.

8. Show that $P_n(1) = 1$ for all n.

9. Define the Hermite polynomials $H_n(x)$ and obtain $H_0(x)$ and $H_1(x)$.

10. Show that $J_1^1(x) = J_0(x) - x^{-1} J_1(x)$.

Part - B (5 × 5 = 25)

Answer **all** questions.

11. (a) State and prove the Fundamental Lemma of calculus of variations.

Or

(b) Find the extremals of the functional

$$v[y(x)] = \int_{x_0}^{x_1} \frac{\sqrt{1+y^{12}}}{y}\, dx.$$

12. (a) Explain the method of solving variational problem in parametric form.

Or

(b) Find the extremals of the functional

$$v[y(x)] = \int_{x_0}^{x_1} \left[(y'')^2 - 2(y')^2 + y^2 - 2y \sin x \right] dx$$

13. (a) Derive the differential equation of free vibrations of a string.

Or

(b) State and prove the Hamilton's principle.

14. (a) Prove that $J_n^{1}(x) = \frac{1}{2}\left[J_{n-1}(x) - J_{n+1}(x)\right]$.

Or

(b) Prove the relation

$$(n+1)\, P_{n+1}(x) = (2n+1)\, x\, P_n(x) - n\, P_{n-1}(x).$$

15. (a) Show that $(1 - 2xt + t^2)^{-1/2} \sum_{n=0}^{\infty} t^n\, P_n(x)$.

Or

(b) Prove that $H_n(x) = (-1)^n e^{x^2} \frac{d^n}{dx^n}\left[e^{-x^2}\right]$.

Part - C (3 × 10 = 30)

Answer any **three** questions.

16. Find the extremals of the isoperimetric problem

$$v\ [\mathrm{y}\ (\mathrm{x})] = \int_0^1 (x^2 - y^{1^2})\ dx \text{ given that}$$

$$\int_0^1 y^2\ dx = 2, y\ \ (0)\ = 0,\ y(1) = 0.$$

17. Find the extremals of the functional

$$v\ (y, z) = \ (\int_{x_0}^{x_1} (2yz - 2y^2 + y^{1^2} - z^{1^2})\ dx.$$

18. Find an approximate solution of the problem of the minimum of the functional

$$v\,[y\,(x)] = \int_0^1 \left[\left(y^1 \right)^2 - y^2 - 2xy \right] dx\,; \quad y\,(0) = y\,(1) = 0 \text{ and}$$

compare it with the exact solution.

19. Prove that $\int_{-1}^{1} P_m\,(x)\,P_n\,(x)\,dx = \begin{cases} 0, & m \neq n \\ \dfrac{2}{2n+1}, & m = n \end{cases}$.

20. Show that

(i) $J_n\,(x) = \dfrac{1}{\pi} \int_0^{\pi} \cos\,(n\,\theta - x \sin\theta)\,d\theta,\ n$

being an integer

(ii) $J_0\,(x) = \dfrac{1}{\pi} \int_0^{\pi} \cos\,(x \cos\phi\,)\,d\phi$.

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AF-2767 **MMA2C1**

M.Sc. DEGREE EXAMINATION, NOVEMBER 2010

Second Semester

Mathematics

ALGEBRA—II

(CBCS—2008 onwards)

Time : 3 Hours Maximum : 75 Marks

Part - A (10 × 2 = 20)

Answer **all** the Questions.

1. Prove that the intersection of two subspace of a vector space V is a subspace of V.

2. If S and T are subsets of a vector space V, prove that $L(S \cup T) = L(S) + L(T)$.

3. Define annihilator A (W) of the subspace W of a vector space V. Prove that if $U \subset W$, then $A(U) \supset A(W)$.

4. If $u, v \in \mathrm{V}$ and $\alpha, \beta \in \mathrm{F}$, prove that :

$$(\alpha u + \beta v, \alpha u + \beta v) = \alpha\bar{\alpha}\,(u,u) + \alpha\bar{\beta}\,(u,v) + \bar{\alpha}\beta\,(v,u) + \beta\bar{\beta}\,(v,v)\,.$$

5. Prove that if K is a finite extension of a field F, then every element of K is algebraic over F.

6. Show that the polynomial $x^5 - x$ over a field F of characteristic zero 5 has no multiple roots.

7. Show that the fixed field of a subgroup H of endomorphisms on a field K is subfield of K.

8. State the fundamental theorem of Galois theory.

9. Let V be the vector space of all polynomials in x of degree $n - 1$ over a field F and D be the differentiation operation on V. Find the matrix of D with respect to the basis $1, x, x^2, \ldots, x^{n-1}$.

10. Prove that $T \in A(V)$ is unitary if and only if $TT^* = 1$.

Part - B (5 × 5 = 25)

Answer **all** Questions.

11 (a) Prove that if $v_1, v_2, \ldots, v_n$ are in V, then either they are linearly independent or some v_k is a linear combinations of $v_1, v_2, \ldots, v_{k-1}$.

(*Or*)

(b) Show that any n dimensional vector space V over a field F is isomorphic to F^n.

12. (a) Show that if V finite dimensional and $v \in V$ and v is nonzero, then there is an element $f \in \hat{V}$ such that $f(v) \neq 0$.

(*Or*)

(b) If , $v \in V$, and inner produce space, prove that $|(u,v)| \leq \| u \| \cdot \| v \|$.

13 (a) If $p(x)$ is an irreducible polynomial in F $[x]$ of degree $n \geq 1$, then show that there is an extension E of F such that $[E : F] = n$ in which $p(x)$ has a root.

(*Or*)

(b) Show that a polynomial $f(x) \in F[x]$ has multiple roots if and only if $f(x)$ and $f'(x)$ have a nontrivial common factor.

14 (a) Let K be a normal extension of F and H be a subgroup of G (K, F). Show that $[K : K_H] = o(H)$ and $H = G(K : K_H)$.

(*Or*)

(b) Let $f(x)$ be a polynomial F[x], K its splitting field over F < G (k, F) its Galois group and T any subfield of K containing F. Prove that T is normal extension of F if, and only if, G (K, T) is a normal subgroup of G (K, F).

15 (a) If $\lambda_1, \lambda_2, .. \lambda_n$ in F are distinct characteristics roots of $T \in A(V)$ and $v_1, v_2, \ldots, v_n$ are characteristic roots belonging to $\lambda_1, \lambda_2, \ldots, \lambda_n$ respectively, show that $v_1, v_2, \ldots, v_n$ are linearly independent over F.

(*Or*)

(b) Prove that the linear transformation $T \in A(V)$ is unitary if and only if it is takes an orthonormal basis into an orthonormal basis of V.

Part - C (3 × 10 = 30)

Answer any **three** Questions.

16. Show that if V is a finite dimensional vector space and if $u_1, u_2, \ldots u_m$ form a basis of V. Use this result to prove that if V is an n dimensional vector space, then any linearly independent subset of B has almost n elements.

17. Show that a finite dimensional inner product space has an orthonormal set as basis.

18. Show that any finite extension of a field of characteristic zero is a simple extension.

19. Prove that K is a normal extension of F if an only if K is the splitting field of some polynomial over F.

20. If $T \in A(V)$ has all its characteristic roots in F, then show that there is a basis of V in which the matrix of T is triangular.

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AF-2768 **MMA2C2**

M.Sc. DEGREE EXAMINATION, NOVEMBER 2010

Second Semester

Mathematics

ANALYSIS–II

(CBCS—2008 onwards)

Time : 3 Hours Maximum : 75 Marks

Part - A (10 × 2 = 20)

Answer **all** questions.

1. Define the upper and lower Riemann integrals of f over $[a, b]$

2. Prove that $\underline{\int_a^b} f d\alpha \leq \overline{\int_a^b} f d\alpha$.

3. Define equicontinuous family of functions.

4. Give an example of uniformly bounded sequence which contain no uniformly convergent subsequences.

5. If $E(z) = \sum_{n=0}^{\infty} \frac{2^n}{n!}, z$ complex, prove that E (z + w) = E (z). E (w).

6. Give an example of an orthonormal system of functions.

7. Prove that the set [0,1] is not countable.

8. If $m^*A = 0$ prove tha $m^* (A \cup B) = m^*B$

9. Define convergence in measure of a sequence of measurable function.

10. Define the Labesgue integral of f over a set E.

Part - B (5 × 5 = 25)

Answer **all** questions.

11 (a) If P* is a refinement of P, prove that :

$L(P, t, \alpha) \leq L(P^*, t, \alpha)$ and

$U(P^*, f, \alpha) \leq U(P, f, \alpha)$

(*Or*)

(b) Suppose f is bounded on $[a, b]$, f has only finitely many points of discontinuity on $[a, b]$ and α is continuous at every point at which f is discontinuous, then prove that $f \in R(\alpha)$.

12. (a) Give an example of a sequence of functions for which the limit of the integral is not equal to integral of the limit eventhough both are finite.

(*Or*)

(b) Prove the existence of real continuous function on the real line which is nowhere differentiable.

13 (a) Suppose $\sum C_n$ converges put

$$f(x) = \sum_{n=0}^{\infty} C_n x^n \, (-1 < x < 1)$$. Then prove that

$$\lim_{x \to 1} f(x) = \sum_{n=0}^{\infty} C_n$$

(*Or*)

(b) Prove that $\Gamma(x) = \frac{2^{x-1}}{\sqrt{x}} \Gamma\left(\frac{x}{2}\right) \Gamma\left(\frac{x+1}{2}\right)$.

14 (a) Prove that the interval (a, ∞) is measurable.

(*Or*)

(b) Let E be a measurable set of finite measure, and $\{f_n\}$ a sequence of measurable function which converge to f a.e. on E. Then prove that given $\in > 0$ and $\delta > 0$, there is a set $A \subset E$ with $mA < \delta$ and an N such that for all $x \notin A$ and all $n \geq N$, $|f_n(x) - f(x)| < \in$.

15 (a) Let ϕ and ψ be simple function which vanish outside a set of finite measure. The prove that $\int(a\phi + b\psi) = a\int\phi + b\int\psi$ and if $\phi \geq \psi$ a.e., then $\int\phi \geq \int\psi$.

(*Or*)

(b) State and prove the Bounded convergence theorem.

Part - C $(3 \times 10 = 30)$

Answer any **three** questions.

16. Assume α increase monotonically $\alpha' \in R$ on $[a, b]$. Let f be a bounded real function on $[a, b]$. Prove that $f \in R(\alpha)$ if and only if $f\alpha' \in R$ and in this case $\int_a^b f d\alpha = \int_a^b f(x)\,\alpha'(x)\,dx$

17. If f is a continuous complex function on $[a, b]$, prove that there exist a sequence of polynomials P_n such that $\lim_{n \to \infty} P_n(x) = f(x)$.

18. State and prove the Parseval's theorem..

19. Let E be a given set. Prove that the following five statements are equivalent :

(a) E is measurable.

(b) Given $\in > 0$, there is an open set $O \supset E$ such that $m^* (O \sim E) < \in$.

(c) Given $\in > 0$, there is a closed set $F \subset E$, such that $m^* (E \sim F) = \in$.

(d) There is a G in G_5 with $E \subset G$, $m^*(G \sim E) = 0$.

(e) There is an F in F_σ with $F \subset G$, m^* $(E \sim F) = 0$

20. Let f be defined and bounded on a measurable set E with mE finite. Then prove that :

$\inf\limits_{f \leq \phi} \int\limits_E \phi\ (x)\ dx = \sup\limits_{f \geq \psi} \int\limits_E \psi\ (x)\ dx$ for all sample functions ϕ and ψ if and only if f is measurable.

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AF-2769 **MMA2C3**

M.Sc. DEGREE EXAMINATION, NOVEMBER 2010

Second Semester

Mathematics

NUMERICAL ANALYSIS

(CBCS—2008 onwards)

Time : 3 Hours Maximum : 75 Marks

Part - A (10 × 2 = 20)

Answer **all** questions.

1. Give the steps of the algorithm for the method of steepest descent.

2. Find the spectral radius of the matrix $\begin{pmatrix} 3 & 2 \\ 1 & 4 \end{pmatrix}$.

3. Show that, if $p(x)$ is a polynomial of degree less than k, then $p(x)$ is orthogonal to $P_k(x)$, for $k = 2$ only.

4. Define Legendre polynomials $p_{k+1}(x)$ and find $p_2(x)$ and $p_3(x)$.

5. A curve passes through the points (0, 18), (1, 10), (3, –18) and (6, 90). Find the slope of the curve at $x = 2$.

6. Evaluate $\int_0^6 \frac{dx}{1+x}$ using Trapezoidal rule taking $h = 1$.

7. Find the general solution of the differential equation $\frac{d^3y}{dx^3} - 2\frac{d^2y}{dx^2} - \frac{dy}{dx} + 2y = 0$.

8. Find the general solution of the difference equation

$y_{n+2} - y_{n+1} - y_n = 0, y_0 = 0$ and $y_1 = 1$.

9. State the recursion formula for the Runge-Kutta method or order 2.

10. What do you mean by multistep methods. Name at least two multistep methods.

Part - B (5 × 5 = 25)

Answer **all** questions.

11. (a) Use the steepest descent method to find the maxima and minima of the function
$$F(x_1, x_2) = \frac{x_1^3}{3} + x_2^2 x_1 + 3.$$

Or

(b) Solve for x and y using Newton's method $x^2 + y - 11 = 0$ and $y^2 + x - 7 = 0$, starting with initial approximations $x_0 = 3.5$ and $y_0 = -1.8$.

12. (a) Derive the normal equations for the best c_1^*, c_2^*, when $F(x) = F(x; c_1, c_2) = c_1 e^{c_2} x$.

Or

(b) Let the scalar product

$< g, h > = \int_a^b g(x) h(x) w(x) dx$. Then prove that $P_k(x)$ has k simple real zeroes, all of which lie in the interval (a, b).

13. (a) A curve passes through the points (1, 2), (1. 5, 2.4), (2, 2.7), (2.5, 2.8), (3, 3), (3.5, 2.6) and (4. 2.1). Obtain the area bounded by the curve ; the x-axis, $x = 1$ and $x = 4$. Also, find the volume of solid of revolution got by revolving this area about the x- axis.

Or

(b) Use Simpson's $-\frac{1}{3}$ rule to estimate the value of

$$I = \int_0^1 (1-x^2)^{3/2}\, dx \text{ by taking } h = 0.1.$$

14. (a) Find the general solution of the difference equation $y_{n+2} - 4y_{n+1} + 3y_n = 2^n$, $y_0 = 0, y_1 = 1$.

Or

(b) Use Tailor's series method to find y (0. 1) correct to five decimal places, given $\frac{dy}{dx} = xy + 1, y(0) = 1$.

15. (a) Use finite difference method to solve for $y\ (x)$ given

$$\frac{d^2 y}{dx^2} + y + 1 = 0,$$

$x \in (0,1),\ y\ (0) = 0,\ y\ (1) = 0$, taking $h = \frac{1}{4}$.

Or

(b) Solve the following problem using the shooting method :

$y'' = 2y^3,\ y\ (1) = 1,\ y\ (2) = \frac{1}{2}$, taking $y'\ (1) = 0$ as a first guess.

Part - C (3 × 10 = 30)

Answer any **three** questions.

16. Solve the non-linear system $x^2 + xy^3 = 9$, $3x^2y - y^3 = 4$ using the fixed point iteration method.

17. If $P_k(x)$ is the Legendre polynomial of degree k, then prove that $\int_{-1}^{1} [P_k(x)]^2\, dx = \frac{2}{2k+1}$. Also device the three term recurrence relation satisfied by Legendre polynomials.

18. Construct a rule of the form

$$I(f) = \int_{-1}^{1} f(x)\, dx \equiv A_0 f\left(-\frac{1}{2}\right) + A, f(0) + A_2 f\left(\frac{1}{2}\right)$$

which is exact for all polynomials of degree ≤ 2.

19. Compute $y\ (0.2)$ given $\frac{dy}{dx} + y + xy^2 = 0$, $y\ (0) = 1$, by taking $h = 0.1$ using Runge-Kutta method of order 4, correct to 4 decimals.

20. Determine the value of $y\ (0.4)$ by Milne's method given $\frac{dy}{dx} = xy + y^2$, $y\ (0) = 1$; use Tailor series to get the values of $y\ (0.\ 1)$, $y\ (0.\ 2)$ and $y\ (0.\ 3)$.

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AF-2770 **MMA2C4**

M.Sc. DEGREE EXAMINATION, NOVEMBER 2010

Second Semester

Mathematics

PROBABILITY AND STATISTICS

(CBCS—2008 onwards)

Time : 3 Hours Maximum : 75 Marks

Part - A $(10 \times 2 = 20)$

Answer **all** questions.

1. Let X have the p.d.f. $f(x) = \frac{1}{x^2}, 1 < x < \infty$

$= 0$ elsewhere.

Show that the mean valve of X does not exist.

2. Let X have the p.d.f. $f(x) = \frac{x}{6}, x = 1, 2, 3$

$= 0$ elsewhere.

Determine $E(X^2)$.

3. X and Y have joint p.d.f given by

$$f(x, y) = 2, \ 0 < x < y < 1$$
$$= 0 \quad \text{elsewhere.}$$

Find the conditional density functions.

4. The joint p.d.f. of X and Y is

$$f(x, y) = e^{-y}, \ 0 < x < y < \infty$$
$$= 0 \qquad \text{elsewhere.}$$

Obtain the marginal density functions.

5. Determine the binomial distribution for which the mean is 4 and variance is 3..

6. If the random variable X has a Poisson distribution such that Pr (X = 1) = Pr (X = 2), find Pr (X = 4).

7. Let X have the p.d.f $f(x)$

$$f(x) = \frac{x^2}{9}, 0 < x < 3$$

$$= 0 \quad \text{elsewhere.}$$

Find the p.d.f. of $Y = X^3$.

8. If X has a p.d.f. $f(x) = 6x(1-x), 0 < x < 1$, determine the value of C such that Pr (X < C) = Pr (X > C).

9. State and explain the importance of Central limit theorem in probability theory.

10. Let $\overline{X}$ denote the mean of a random sample of size 100 from a distribution that is $\chi^2(50)$. Find an approximate value of Pr $(49 < \overline{X} < 51)$.

Part - B $(5 \times 5 = 25)$

Answer **all** questions.

11. (a) The probabilities of X, Y and Z becoming managers are $\frac{4}{9}, \frac{2}{9}$ and $\frac{1}{3}$ respectively. The probabilities that a Bonus scheme will be introduced if X, Y and Z becomes managers are $\frac{3}{10}, \frac{1}{2}$ and $\frac{4}{5}$ respectively. What is the probability that the Bonus Scheme will be introduced ?

(*Or*)

(b) If X has the p.d.f. $f(x) = \left(\frac{1}{2}\right)^x, x = 1, 2, 3 \ldots.$

$= 0$ elsewhere,

find the m.g.f. mean and variance of X.

12. (a) Let X and Y have the joint p.d.f.

$$f(x, y) = x + y,\ 0 < x < 1,\ 0 < y < 1.$$

$$= 0 \qquad \text{elsewhere.}$$

Computer the correlation coefficient of X and Y.

(*Or*)

(b) If the random variables X and Y have the joint p.d.f.

$$f(x, y) = 12\,xy\,(1 - y),\ 0 < x < 1,\ 0 < y < 1.$$

$$= 0 \qquad \text{elsewhere.}$$

prove that X and Y are stochastically independent.

13. (a) Compute the measures of SKewness and Kurtosis of a Poisson distribution with parameter μ.

(*Or*)

(b) Let X have a gamma distribution with $\alpha = \frac{r}{2}$, where r is a positive integer, and $\beta > 0$. Prove that the distribution of $Y = \frac{2X}{\beta}$ is $\chi^2(r)$.

14. (a) If X_1, X_2 is a random sample of size 2 from a distribution with p.d.f.

$$f(x) = 2x, \; 0 < x < 1$$
$$= 0 \quad \text{elsewhere,}$$

find $\Pr(X_1 / X_2 \leq ½)$.

(*Or*)

(b) Let X have the p.d.f. $f(x) = 1, 0 < x < 1$

$= 0$ elsewhere.

Prove that $y = 2 \ln X$ has a Chi-square distribution with 2 degrees of freedom.

15. (a) If $\overline{X}$ is the mean of a random sample of size n from a normal distribution with mean μ and variance 100, find n such that

$$\Pr\left(-5 < \overline{X} < +5 \right) = 0.954$$

(*Or*)

(b) Let Y_n be the n^{th} order statistic of a random sample $X_1, X_2 \ldots X_n$ from a distribution with p.d.f.

$$f(x) = \frac{1}{\theta}, 0 < x < 1, \theta < y < \infty.$$

$= 0$ elsewhere.

Find the limiting distribution of Y_n.

Part - C (3 × 10 = 30)

Answer any **three** questions.

16. (a) Establish Chebyshev's inequality and give its importance.

(b) Let X have a Poisson distribution with $\mu = 100$. Find a lower bound for Pr $(75 < X < 125)$ using Chebyshev's inequality.

17. (a) If the correlation coefficient ρ of X and Y exists, show that $-1 \leq \rho \leq 1$.

(b) Joint distribution of the random variables X and Y is

$$f(x, y) = 4xy,\ e^{-(x^2 + y^2)}\ ; x > 0, y > 0.$$

Examine whether X and Y are independent.

18. (a) Derive the Poisson distribution as the limiting form of binomial distribution.

(b) Show that the mean, median and mode coincide for normal distribution.

19. Derive the p.d.f. of F-distribution and obtain its mode.

20. State and prove any two theorems on limiting distributions.

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AF-2771 **MMA2E1**

M.Sc. DEGREE EXAMINATION, NOVEMBER 2010

Second Semester

Mathematics

Elective—APPLIED ALGEBRA

(CBCS—2008 onwards)

Time : 3 Hours Maximum : 75 Marks

Part - A $(10 \times 2 = 20)$

Answer **all** questions.

1. Define a finite-state machine.

2. Define :

   (i) Covering machines and

   (ii) Equivalent machines.

3. What is meant by type declaration ? Give an example.

4. Evaluate the following ALGOL expressions when $A = 2, B = 3, C = -2, D = 4, E = 5, F = 6$ and $G = -2$.

   (i) $A \times D - E + F \uparrow G$.

   (ii) $A - B \times C + D$

5. Examine whether the following is a tautology of a Boolean algebra B : $(p \vee q) \rightarrow p$

   Justify your answer.

6. Given two Boolean algebras A and B, define their direct product $A \times B$.

7. Prove that any subpath of an optimal path is optimal

8. Give an example of a flip-flop.

9. Explain the terms 'encoding scheme' and 'decoding scheme'.

10. What is a group code ? Give an example

Part - B (5 × 5 = 25)

Answer **all** questions.

11. (a) Draw the state diagram for the following automation :

	γ		ξ	
	0	1	0	1
1	1	2	0	0
2	2	3	0	0
3	3	4	0	0
4	4	1	0	1

Or

(b) Let M, $\overline{\mathrm{M}}$ be finite state machines. Prove that every epimorphism $\mu : \mathrm{M} \to \overline{\mathrm{M}}$ defines a covering of $\overline{\mathrm{M}}$ by M.

12. (a) Write a program segment in ALGOL to generate array K with K $[i] = i\,!$ for $i = 1, 2, ..., 10$.

Or

(b) If A, B are both 3 × 3 real matrices, write an ALGUL program to find the matrix product AB.

13.(a) In any Boolean algebra, prove that the following four equations are mutually equivalent :

$$a \wedge b = a$$
$$a \vee b = b$$
$$a' \vee b = \mathrm{I}$$
$$\mathrm{a} \wedge b' = 0$$

Or

(b) Prove that any interval $[a, b]$ of a Boolean algebra A is a distributive lattice under the operations of A.

14. (a) Prove that any Boolean function of n variables can be expressed in product of sums form.

Or

(b) Find a minimal sum of -products expression for

$$f(a, b, c) = \sum (0, 3, 4, 6)$$

15. (a) For a code to detect all sets of k or fever errors, prove that it is necessary and sufficient that the minimum distance between any two code words be $k + 1$ or more.

Or

(b) What is a Hamming code ? How is it formed ?

Part - C $(3 \times 10 = 30)$

Answer any **three** questions.

16. Find the minimal machine equivalent to the following machine :

	γ		ξ	
	a	b	a	b
1	1	6	0	0
2	1	4	0	0
3	2	5	1	0
4	5	8	1	1
5	1	3	1	0
6	8	5	1	1
7	6	3	1	1
8	2	5	1	0

17. Illustrate the concepts of a block and a compound statement in ALGOL with suitable examples. Explain the difference between them.

18. (a) Let $\mathcal{B}$ denote a Boolean algebra. Prove that every function $f : \mathcal{B}^n \to \mathcal{B}$ is a Boolean combination of the co-ordinate functions B

$\delta_i : \vec{x} \to x_i$, this combination being defined by some Boolean polynomial $f(\delta_1, ..., \delta_n)$

(b) In any Boolean algebra, prove the following : If $a \wedge x = a \wedge y$ and $a \vee x = a \vee y$, then $x = y$.

19. (a) Explain the procedure for deriving prime implicants.

(b) Apply the procedure to the canonical sum-of-products expression

$$\phi = a\,b\,c\,d \vee a\,b'\,c\,d' \vee a\,b'\,c\,d \vee a\,b\,c\,d' \vee a'\,b'\,c\,d'$$

20. (a) Describe a systematic algebraic technique for encoding binary words by matrix multiplication.

(b) Describe the complete (3, 6) encoding defined by the matrix.

$$\mathrm{E} = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 \end{bmatrix}$$

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AF-2772 **MMA2E2**

M.Sc. DEGREE EXAMINATION
NOVEMBER 2010

Second Semester

Mathematics

Elective—PROGRAMMING IN JAVA

(CBCS—2008 onwards)

Time : 3 Hours Maximum : 60 Marks

Part - A ($10 \times 1\frac{1}{2} = 15$)

Answer **all** questions.

1. Define Package.

2. Give some examples for runnable interface.

3. List various built-in exceptions.

4. Write the advantages of swing over AWT.

5. Define Method Overriding.

6. What is meant by data encapsulation ? Illustrate with an example.

7. Why Wrapper classes are used in Java ?

8. When do we design an "Interface" in Java ?

9. List various built–in exceptions.

10. What is meant by Garbage Collection ? How it is used in Java ?

Part - B

(5 × 3 = 15)

Answer **all** questions.

11. (a) Explain the "loop" control structures available in Java.

(*Or*)

(b) Write a simple Java program to sort a given 'n' number of names.

12. (a) What are the various actions performed using Button ? Explain.

(*Or*)

(b) Write a Java program to throw an exception when the numeric input is less than zero.

13. (a) What are the various states in thread ? Explain.

(*Or*)

(b) Explain the use of nested "try" Statements.

14. (a) What do you mean by "Applet" ? Write a simple program using applets.

(*Or*)

(b) What are the issues of designing Interface in Java ?

15. (a) Write a Java program to receive two values as command line arguments and display their sum.

(*Or*)

(b) Differentiate Servlet and Javascript.

Part - C $(3 \times 10 = 30)$

Answer any **three** questions.

16. Write a Java program to get a string and to reverse it and display.

17. What is the difference between Method overloading and Method overriding ? Explain it with suitable examples.

18. List four differences between a Java application program and Java applet program with an example of each type of program.

19. Define various access specifiers in Java. And describe multiple inheritance in Java.

20. What is meant by super keyword in Java ? Explain it with a simple program in Java.

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AF-2773

MMA2E3

M.Sc. DEGREE EXAMINATION, NOVEMBER 2010

Second Semester

Mathematics

Elective—CODING THEORY

(CBCS—2008 onwards)

Time : 3 Hours — Maximum : 75 Marks

Part - A — ($10 \times 2 = 20$)

Answer **all** questions.

1. Show that any q-ary (n, q, n) - code is equivalent to a repetition code.

2. Show that the number of inequivalent binary codes of length n and containing just two code words is n.

3. Define a linear code and give an example.

4. Define equivalent linear codes and give an example.

5. What are perfect codes ? Give examples.

6. Is the code G_{24} self dual ? Justify.

7. Define a cyclic code and give an example.

8. Define a binary Golay code and give an example.

9. Define a BCH code. Give an example.

10. Let C be the linear [10, 6]-code over GF (11) defined to have parity check matrix $H = \begin{bmatrix} 1 & 1 & 1 & \ldots & 1 \\ 1 & 2 & 3 & \ldots & 10 \\ 1^2 & 2^2 & 3^2 & \ldots & 10^2 \\ 1^3 & 2^3 & 3^3 & \ldots & 10^3 \end{bmatrix}$

Decode the received vector 1204000910, using the above code.

Part - B $(5 \times 5 = 25)$

Answer **all** questions.

11 (a) If d is even and $n < 2d$, prove that $A_2(n, d) \leq 2\,[d/(2d - n)]$ and if d is odd and $n < 2d + 1$, prove that $A_2(n, d) \leq 2\,[(d + 1)/(2d + 1 - n)]$.

(*Or*)

(b) Prove that (i) a code C can detect upto s errors in any code word if $d(c) \geq s + 1$ and (ii) a code C can correct upto r errors in any code if $d(c) \geq 2r + 1$.

12. (a) Let C be the ternary linear code with generator matrix $\begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 2 \end{bmatrix}$. List the code words of C and find the minimum distance of C.

(*Or*)

(b) What are the advantages and disadvantages of linear codes ?

13 (a) Does there exist a 6-ary $(7, 6^5, 3)$ - code ? Justify.

(*Or*)

(b) Does there exist a binary linear [90, 78, 5]-code ? Justify.

14 (a) Obtain a necessary and sufficient condition for a code C in R_n to be a cyclic code.

(*Or*)

(b) Find all the ternary cyclic codes of length 4 and write down a generator matrix for each of them.

15 (a) Suppose we want to give each person in a population of some 2,00,000 a personal identity code word composed of letters of the English alphabet. Devise a suitable code of reasonably short length which is double-error-correcting.

(*Or*)

(b) Suppose $a_1, a_2, a_3, \ldots, a_n$ are distinct non-zero elements of a field. Prove that the matrix

$$A = \begin{bmatrix} 1 & 1 & \cdots\cdots\cdots & 1 \\ a_1 & a_2 & \cdots\cdots\cdots & a_n \\ a_1^2 & a_2^2 & \cdots\cdots\cdots & a_n^2 \\ \vdots & & \cdots\cdots\cdots & \\ \vdots & & \cdots\cdots\cdots & \\ a_1^{n-1} & a_2^{n-1} & \cdots\cdots\cdots & a_n^{n-1} \end{bmatrix}$$

Part - C $(3 \times 10 = 30)$

Answer any **three** questions.

16. (a) If there exists a binary (n, M, d) - code, prove that there exists a binary $(n-1, M', d)$ code with $M' \geq M/2$. Deduce that $A_2(n, d) \leq 2A_2(n-1, d)$.

(b) Show that $A_2(8, 5) = 4$ and that upto equivalence there is just one binary (8, 4, 5) - code.

17. (a) In a binary linear code, prove that either all the code-words have even weight or exactly half have even weight and half have odd weight.

(b) Suppose $[I_k, A]$ is a standard form generator matrix for a linear code C. Prove that any permutation of the rows of A gives a generator matrix for a code which is equivalent to C.

18. Prove that the code G_{24} having the generator matrix $G = [l_{12} | A]$ is a [24, 12, 8]-code.

19. (a) Suppose p is an odd prime number and that over GF (2), $x^p - 1 = (x - 1)\, g_1(x).\ \overline{g_1(x)}$. Let $a(x)$ be a code word of $< g(x) >$ of even weight w. Then, prove (i) $w \equiv 0 (\bmod 4)$ and (ii) $w \neq 4$ unless $p = 7$.

(b) Suppose C is a cyclic code with generator polynomial $g(x) = g_0 + g_1 x + \ldots + g_r x^r$ of degree r. Prove that dim (C) $= n - r$ and find the generator matrix for C.

20. (a) Explain the error-correction procedure.

(b) Consider the 3-error-correcting code over GF(11) with the parity check matrix

$$\begin{bmatrix} 1 & 1 & 1 & \dots & 1 \\ 1 & 2 & 3 & \dots & 10 \\ 1^2 & 2^2 & 3^2 & \dots & 10^2 \\ 1^3 & 2^3 & 3^3 & \dots & 10^3 \\ 1^4 & 2^4 & 3^4 & \dots & 10^4 \\ 1^5 & 2^5 & 3^5 & \dots & 10^5 \end{bmatrix}$$

Suppose we have received a vector whose syndrome has been calculated to be $(S_1, S_2, S_3, S_4, S_5, S_6) = (2, 8, 4, 5, 3, 2)$. Assuming at most 3 errors, find the error vector.

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AF-2774 **MMA3C1**

M.Sc. DEGREE EXAMINATION, NOVEMBER 2010

Third Semester

Mathematics

COMPLEX ANALYSIS

(CBCS—2008 onwards)

Time : 3 Hours Maximum : 75 Marks

Part - A (10 × 2 = 20)

Answer **all** questions.

1. Define convergent sequence and prove that a convergent sequence is bounded.

2. Find the radius of convergence of the power series $\sum \frac{z^n}{n!}$.

3. Prove that $\int_{\nu} f(z)\, dz$, with continuous f, depends only on the end points of ν if and only if f is the derivative of an analytic function in Ω.

4. Compute $\int_{\nu} x\, dz$ where ν is the directed line segment from 0 to $1 + i$.

5. Show that the function $\sin z$ has an essential singularity at ∞.

6. Prove that $f(z) = \frac{\sin z}{z}$ has a removable singularity at $z = 0$.

7. Find the residue of $f(z) = \dfrac{z^2}{(z-1)^2 (z+2)}$ at $z = -2$.

8. Evaluate $\displaystyle\int_{|z|=1} \frac{z-3}{z^2 + 2z + 5}\, dz$.

9. If $f(z)$ is an entire function which is never zero, prove that $f(z) = e^{g(z)}$ where $g(z)$ is an entire function.

10. Expand $\dfrac{z-1}{z+1}$ in Taylor's series about the point $z = 0$.

Part - B (5 × 5 = 25)

Answer **all** questions.

11. (a) Show that an analytic function cannot have a constant absolute value without reducing to a constant.

Or

(b) Find the linear transformation which carries $0, i, -i$ into $1, -1, 0$.

12. (a) Prove that the line integral $\int_{\nu} p\,dx + q\,dy$, defined in Ω, depends only on the end points of ν if and only if there exists a function $u\,(x, y)$ in Ω with the partial derivatives $\frac{\partial U}{\partial x} = p, \frac{\partial U}{\partial y} = q.$

Or

(b) If the piecewise differentiable closed curve γ does not pass through the point a, prove that the value of the integral $\int_{\nu} \frac{dz}{z-a}$ is an integral multiple of $2\,\pi\,i$.

13. (a) State and prove the Weierstrass theorem on analytic functions.

Or

(b) If $f(z)$ is analytic and non-constant ina region Ω, prove that its absolute value $|f(z)|$ has no maximum in Ω.

14. (a) State and prove the Argument principle.

Or

(b) Compute $\int_0^{\pi} \frac{d\theta}{17 - 8\cos\theta}$.

15. (a) State and prove the Hurwitz theorem.

Or

(b) Prove that for $|z| < 1$,

$$(1+z)(1+z^2)(1+z^4)(1+z^8)\ldots\ldots = \frac{1}{1-z}.$$

Part - C (3 × 10 = 30)

Answer any **three** questions.

16. State and prove the necessary and sufficient condition for the function $w = f(z)$ to be analytic.

17. Suppose that $\phi(\varepsilon)$ is continuous on the arc γ. Prove that the function $F_n(z) = \int_{\nu} \frac{\phi(\varepsilon)\, d\varepsilon}{(\varepsilon - z)^n}$ is analytic in each of the regions determined by ν, and that its derivative is given by $F_n^{1}(z) = n\, F_{n+1}(z)$.

18. State and prove the local mapping theorem.

19. Evaluate

(i) $\int_{0}^{\pi/2} \frac{dx}{a + \sin^2 x}$; $|a| > 1$ and

(i) $\int_{0}^{\infty} \frac{x^2\, dx}{(x^2 + a^2)^2}$, a real.

20. State and prove the Mittag-Leffler's theorem.

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AF-2775 MMA3C2

M.Sc. DEGREE EXAMINATION, NOVEMBER 2010

Third Semester

Mathematics

TOPOLOGY—I

(CBCS—2008 onwards)

Time : 3 Hours Maximum : 75 Marks

Part - A $(10 \times 2 = 20)$

Answer **all** questions.

1. Define a topology on a set X and give an example.

2. Define a Hausdorff space and give an example.

3. Define a homeomorphism and give an example.

4. Define box topology.

5. Define a path connected space.

6. Let A be a connected subset of a topological space X. If $A \subset B \subset \overline{A}$, then prove that B is also connected.

7. Define a compact space and give an example.

8. State tube Lemma.

9. Define a regular space and give an example.

10. State Tietz extension Theorem.

Part - B (5 × 5 = 25)

Answer **all** questions.

11. (a) If $\mathscr{B}$ is a basis for the topology of X and $\mathscr{C}$ is a basis for the topology of Y, then prove that the collection.

$\mathscr{D} = \{B \times C / B \in \mathscr{B} \text{ and } C \in \mathscr{C}\}$ is a basis for the topology of XxY.

(*Or*)

(b) Let A be a subset of a topological space X and Let $\overline{A}$ denote the closure of A in X. Prove that $x \in \overline{A}$ if and only if every open set $\cup$ containing x intersects A.

12. (a) State and prove the pasting lemma.

(*Or*)

(b) State and prove the sequence lemma.

13. (a) Prove that continuous image of a connected space is connected.

(*Or*)

(b) If X is a topological space, then prove that each path component of X lies in a component of X. Also prove that if X is locally path connected, then the components and path components of X are the same.

14. (a) Prove that every compact subset a Hausdorff space is closed.

(*Or*)

(b) Prove that in a topological space compactness implies limit point compactness.

15. (a) Suppose that a space X has a countable basis, then prove that every open covering of X contains a countable subcollection covering X.

(*Or*)

(b) Prove that every metrizable space is normal.

Part - C (3 × 10 = 30)

Answer any **three** questions.

16. (a) Let $\mathscr{B}$ and $\mathscr{B}'$ be bases for the topologies τ and τ' respectively, on X. Then prove that the following statements are equivalent :

(i) τ' is finer that τ.

(ii) For each $x \in X$ and each basis element $B \in \mathscr{B}$ containing x, there is a basis element $B' \in \mathscr{B}'$ such that $x \in B' \subset B$.

(b) Let Y be a subspace of a topological space X. Then prove that a set A is closed in Y iff it equals the intersection of a closed set of X with Y.

17. Let X and Y be topological spaces. Let $f : X \to Y$. Then prove that the following statements are equivalent :

(i) f is continuous.

(ii) For every subset A of X, $f(\overline{A}) \subset \overline{f(A)}$.

(iii) For every closed set B in Y, the set $f^{-1}(B)$ is closed in X.

18. Prove that the Cartesian product of connected spaces is connected.

19. Let X be a set having the least upper bound properly. Prove that in the order topology, each closed interval in X is compact.

20. State and prove Urysnon Metrization Theorem.

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AF-2776 | **MMA3C3**

M.Sc. DEGREE EXAMINATION, NOVEMBER 2010

Third Semester

Mathematics

GRAPH THEORY

(CBCS—2008 onwards)

Time : 3 Hours | Maximum : 75 Marks

Part - A (10 × 2 = 20)

Answer **all** questions.

1. If $G \cong H$, then $\gamma(G) = \gamma(H)$ and $\varepsilon(G) = \varepsilon(H)$. Give an example to show that the converse is false.

2. Show that $\delta \leq \frac{2\varepsilon}{\gamma} \leq \Delta$.

3. Compute K, K' and δ for the complete bipartite graph $K_{n,n}$.

4. Explain Konigsberg bridge problem.

5. Let M be a matching in G. Define an M-augmenting path.

6. State Konig's theorem.

7. Prove that $r\,(2, l) = l$.

8. Prove that a graph is 2-colorable if and only if it contains no odd cycles.

9. Let H be a given subgraph of a graph G. Define a bridge of H in G.

10. Find a planar embedding of $K_5 - e$.

Part - B $(5 \times 5 = 25)$

Answer **all** questions.

11. (a) Prove that a vertex v of a tree G is a cut vertex of G if and only if $d(v) > 1$.

(*Or*)

(b) Let T be a spanning tree of a connected graph G, and let e be any edge of T. Define a cotree and prove that the cotree $\overline{T}$ contains no bond of G.

12. (a) Prove that $K' \leq \delta$ and find a simple graph G with $K = 2$, $K' = 3$ and $\delta = 4$.

(*Or*)

(b) If G is Hamiltonian, then prove that for every nonempty proper subset S of V, $\omega(G-S) \leq |S|$.

13. (a) Let M be a matching in G. Prove that if G contains no M-augmenting path, then M is a maximum matching.

(*Or*)

(b) If G is a k-regular bipartite graph with $k>0$, then prove that G has a perfect matching.

14. (a) Define an independent set and a covering. Prove that a set $S \subseteq V$ is an independent set of G if and only if $V | S$ is a covering of G.

(*Or*)

(b) Prove that in a critical graph, no vertex cut is a clique.

15. (a) If G is a connected plane graph, then prove that $\gamma - \varepsilon + \varphi = 2$.

(*Or*)

(b) Show that every planar graph is 6 - vertex colourable.

Part - C (3 × 10 = 30)

Answer any **three** questions.

16. Prove that $\tau(K_n) = n^{n-2}$.

17. Let G be a simple graph with degree sequence $(d_1, d_2, \dots d_\gamma)$, where $d_1 \le d_2 \le \dots \le d_\gamma$ and $\gamma \ge 3$. Suppose that there is no value of m less then $\frac{\gamma}{2}$ for which $d_m \le m$ and $d_{\gamma - m\cdot} < \gamma - m$. Then prove that, G is Hamiltonian.

18. Prove that if G is simple, then either $\chi' = \Delta$ or $\chi' = \Delta + 1$.

19. State and prove Brooks' theorem.

20. Prove that the following three statements are equivalent :

    (i) every planar graph is 4–vertex–colourable;

    (ii) every plane graph is 4–face–colourable;

    (iii) every simple 2 – edge – connected 3 – regular planar graph is 3 – edge colourable.

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AF-2777 **MMA3C4**

M.Sc. DEGREE EXAMINATION, NOVEMBER 2010

Third Semester

Mathematics

OPERATIONS RESEARCH

(CBCS—2008 onwards)

Time : 3 Hours Maximum : 75 Marks

Part - A $(10 \times 2 = 20)$

Answer **all** questions.

1. State the advantages and disadvantages of network analysis.

2. Define total float and free float.

3. What are the costs associated with inventory ?

4. Explain the terms lead time and re-order point.

5. What are the limitations of Queueing theory ?

6. Explain the terms Balking and Jockeying.

7. Explain the single channel Queueing model.

8. Write formula for finding

L_s in $(M / G / 1) : (G D / \infty / \infty)$ (P – K formula) model.

9. Explain dichotomous search method.

10. What do you mean by separable programming ?

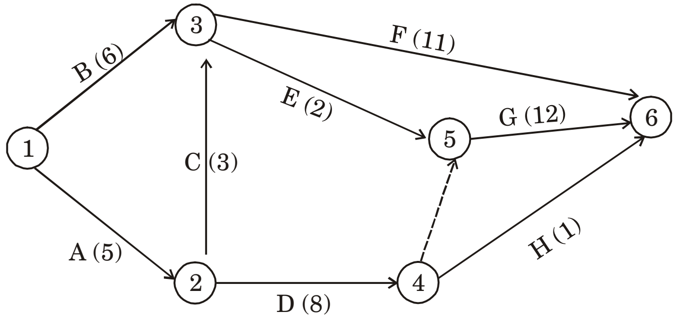
Part - B $(5 \times 5 = 25)$

Answer **all** questions.

11. (a) Explain the minimal spanning tree algorithm.

(*Or*)

(b) Determine the critical path for the project network.



12. (a) Explain the EOQ system with Price Breaks.

(*Or*)

(b) The following data describe four inventory items. The company wishes to determine the economic order Quantity for each of the four items such that the total number of orders per year (365 days) is atmost 150.

Item i	k_i (\$)	D_i (units/d)	h_i (\$)
1	100	10	0.1
2	50	20	0.2
3	90	5	0.2
4	20	10	0.1

Solve the Problem.

13. (a) Describe the basic elements of a Queueing model.

(*Or*)

(b) Babies are born in a sparsely populated state at the rate of one birth for every 12 minutes. The time between births follows an exponential distribution. Find the following :—

(i) The average number of births per year.

(ii) The probability that no births will occur in any one day.

(iii) The probability of issuing 50 birth certificates by the end of the next 3 hours given that 40 certificates were issued during the last 2 hours.

14. (a) Describe the transition - rate diagram for finding probabilities p_n in the Queueing system.

(*Or*)

(b) Patients arrive at a clinic according to a Poisson distribution at the rate of 20 patients per hour. The waiting room does not accommodate more than 14 patients. Examination time per patient is exponential, with a mean of 8 minutes.

(i) What is the probability that an arriving patient will not wait ?

(ii) What is the expected waiting time until a patient leaves the clinic ?

15. (a) Find the maximum of the function

$$f(x) = \begin{cases} 3x, & 0 \le x \le 2 \\ \dfrac{-x}{3} + \dfrac{20}{3} & 2 \le x \le 3, \end{cases}$$

by dichotomous search. Assume that $\Delta = 0.001$.

(*Or*)

(b) Solve the following separable programming problem :

$$\text{Minimize } Z = x_1^2 + x_2^2 + 5$$

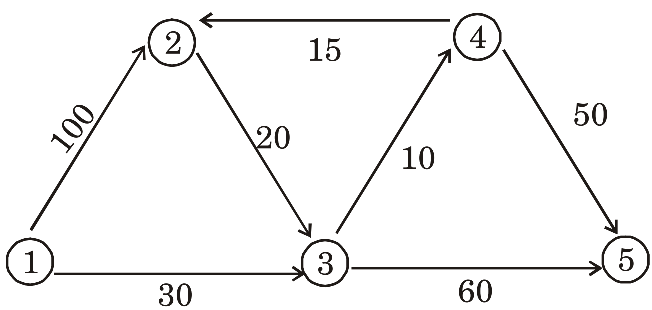
subject to

$$3x_1^4 + x_2 \le 243$$
$$x_1 + 2x_2^2 \le 32$$
$$x_1, x_2 \ge 0$$

Part - C (3 × 10 = 30)

Answer any **three** questions.

16. The following network gives the permissible routes and their lengths in miles between city 1 (node 1) and four other cities (nodes 2 to 5). Determine the shortest routes from city 1 to each of the remaining four cities.



17. Neon lights on the Uof A campus are replaced at the rate of 100 units per day. The physical plant orders the neon lights periodically. It costs \$ 100 to initiate a purchase order. A neon light kept in storage is estimated to cost about \$ 0.02 per day. The lead time between placing and receiving an order is 12 days. Determine the optimal inventory policy for ordering the neon lights.

18. Explain the pure Birth and pure death models in Queueing theory.

19. B and K Groceries operates with three check-out counters. The sign by the check out area advises the customers that an additional counter will be opened any time the number of customers in any lane exceed three. This means that for fever than four customers, only one counter will be in operation. For four to six customers, two counters will be open. For more than six customers, all three counters will be open. The customers arrive at the counters area according to a poisson distribution, with a mean of 10 customers per hour. The average check out time per customer is exponential with mean 12 minutes. Determine the steady state probability p_n of n customers in the check - out area.

20. Illustrates the application of the restricted basis method with the following separable programming problem :

Maximize $Z = x_1 + x_2^4$ subject to

$$3x_1 + 2x_2^2 \le 9,$$
$$x_1, x_2 \ge 0.$$

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AF-2778 **MMA3E1**

M.Sc. DEGREE EXAMINATION, NOVEMBER 2010

Third Semester

Mathematics

Elective—COMBINATORIAL MATHEMATICS

(CBCS—2008 onwards)

Time : 3 Hours Maximum : 75 Marks

Part - A (10 × 2 = 20)

Answer **all** Questions.

1. Show that $n \times (n - 1, r) = r + 1)$.

2. Among the 10,000 numbers between 1 and 10,000 how many of them contain the digit 1 ? How many of them do not ?

3. Find the coefficient of the term x^{28} in $(1 + x^5 + x^9)^{100}$.

4. Solve the difference equation $a_n + 2a_{n-1} + a_{n-2} = 2^n$.

5. In how many ways can the integers 1, 2, 3, 4, 5, 6, 7, 8, 9 be permuted such that no odd integer will be in its natural position ?

6. Find the number of permutations of the letters a, b, c, d, e, and f in which neither the pattern ace nor the pattern f_d appears.

7. Find the cycle index of the group :

$$G = \left\{ \begin{pmatrix} a & b & c & d \\ a & b & c & d \end{pmatrix}, \begin{pmatrix} a & b & c & d \\ b & c & a & d \end{pmatrix}, \begin{pmatrix} a & b & c & d \\ c & a & b & d \end{pmatrix} \right\}.$$

8. Let D = $\{a, b, c, d\}$, R = $\{x, y\}$. Let G be the permutation group

$$\left\{ \begin{pmatrix} a & b & c & d \\ b & c & d & a \end{pmatrix}, \begin{pmatrix} a & b & c & d \\ c & d & a & b \end{pmatrix}, \begin{pmatrix} a & b & c & d \\ d & a & b & c \end{pmatrix}, \begin{pmatrix} a & b & c & d \\ a & b & c & d \end{pmatrix} \right\}$$

If $f_2(a) = y$, $f_2(b) = x$, $f_2(c) = x = f_2(d)$ and $f_4(a) = f_4(b) = x$, $f_4(c) = y$, $f_4(d) = x$, check whether f_2 and f_4 are equivalent.

9. What is a block design ? Give an example.

10. Define a normal matrix and give an example.

Part - B $(5 \times 5 = 25)$

Answer **all** Questions.

11. (a) Find the number of n-digit words generated from the alphabet $\{0, 1, 2, 3, 4\}$ in each of which the total number of 0's and 1's is even.

(*Or*)

(b) In how many ways can three numbers be selected from the numbers $1, 2, \ldots, 300$ such that their sum is divisible by 3 ?

12. (a) Solve the difference equation $a_n + 2a_{n-1} = n + 3$.

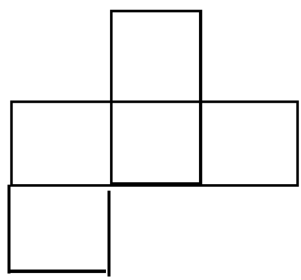
(*Or*)

(b) A circle and n straight lines are drawn on a plane. Each of these lines intersects all the other lines inside the circle. If no three or more lines meet at one point into how many regions do these lines divide the circle ?

13. (a) Find the number of n-digit ternary sequences that have an even number of 0's.

(*Or*)

(b) Find the root polynomial of the chess board



14. (a) Find the distinct ways of painting the eight vertices of a cube with two colours x and y.

(*Or*)

(b) Find the number of distinct bracelets of five beads made up of yellow, blue and white beads.

15. (a) Given a finite projective Geometry PG (n, q), prove that a (v, k, λ) design can be obtained by considering the distinct points of geometry as objects and the distinct hyperplanes as blocks.

(*Or*)

(b) If $n = p_1^{\alpha_1} . p_2^{\alpha_2} \dots p_k^{\alpha_k}$ is the prime factorization of n and if $v(n) = \min\left(p_i^{\alpha_i} - 1\right)$, prove that there exist $v(n)$ Mutually orthogonal latin squares of order n.

Part - C (3 × 10 = 30)

Answer any **three** Questions.

16. Evaluate the sums : (i) $1^2 + 2^2 + 3^2 + \dots + r^2$

(ii) $\sum_{i=0}^{r} \frac{r!}{(r-i+1)!\,(i+1)!}$.

17. Among the 4^n n - digit quaternary sequences, how many of them have an even number of 0's and an even number of 1's ?

18. State and prove the principle of Inclusion and Exclusion.

19. State and prove Polya's fundamental theorem.

20. State and prove Ryser theorem of symmetric designs.

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AF-2779

MMA3E2

M.Sc. DEGREE EXAMINATION, NOVEMBER 2010

Third Semester

Mathematics

Elective—STOCHASTIC PROCESSES

(CBCS—2008 onwards)

Time : 3 Hours　　　　Maximum : 75 Marks

Part - A　　(10 × 2 = 20)

Answer **all** questions.

1. Define Stochastic processes.

2. Define Markov process.

3. Define Transition probability matrix.

4. Define the Recurrent state of a Markov chain.

5. Define Aperiodic state of a Markov chain.

6. Define Poisson process.

7. Define Renewal process.

8. Define Irreducible Markov chain.

9. Define Birth and Death process.

10. State Chapman-Kolmogorov equation.

Part - B $(5 \times 5 = 25)$

Answer **all** questions.

11 (a) Find the mean and variance of the following probability distribution :

$$\Pr\{X(t) = n\} = \frac{(at)^{n-1}}{(1+at)^{n+1}} \ ; n = 1, 2, 3 \ldots$$

$$= \frac{at}{1+at}, \ n = 0$$

(*Or*)

(b) Find the covariance function of $\{Y_n, n \geq 1\}$ given by $Y_n = a_0 X_n + a_1 X_{n-1} + \ldots + a_k X_{n-k}$, $n = 1, 2, \ldots$

where a's are constants and X_n's are uncorrelated random variables.

12. (a) The t.p.m. of a Markov chain $\{X_n, n = 1, 2, \ldots\}$ having three states 1,2 and 3 is :

$$P = \begin{bmatrix} 0.1 & 0.5 & 0.4 \\ 0.6 & 0.2 & 0.2 \\ 0.3 & 0.4 & 0.3 \end{bmatrix}$$

and the initial distribution is (0.7, 0.2, 0.1).

Find $P\{X_3 = 2, X_2 = 3, X_1 = 3, X_0 = 2\}$.

(*Or*)

(b) Three boys A, B and C are throwing a ball to each other. A always throws the ball to B and B always throws the ball to C, but C is just as likely to throw the ball to B as to A. Show that the process is Markovian. Find the transition matrix and also classify the states.

13 (a) State and prove any two properties of Poisson process.

(*Or*)

(b) For the pure birth process show that the interval T_k between the K^{th} and $(K+1)^{st}$ birth has an exponential distribution with parameter λ_k.

14 (a) The number of accidents in a town follows a Poisson process with a mean of 2 per day and the number X_i of people involved in the i^{th} accident has the distribution (independent)

$$\Pr\{X_i = k\} = \frac{1}{2^k} (k \geq 1).$$

Find the mean and variance of the number of people involved in accidents per week.

(*Or*)

(b) If customers arrive at a counter in accordance with a Poisson process with a mean rate 2 per minute, find the probability that the interval between 2 consecutive arrivals is (i) more than 1 min (ii) between 1 min and 2 min and (iii) 4 min (or) less.

15 (a) State and prove Fundamental Renewal equation.

(*Or*)

(b) A student's study habits are as follows :

If he studies, one night, he is 70% sure not to study the next night. On the other hand, if he does not study one night he is 60% sure not to study the next night as well. In the long run, how often does he study ?

Part - C (3 × 10 = 30)

Answer any **three** questions.

16. Assume that a computer system is in one of three states : busy, idle (or) undergoing repair, respectively denoted by states 0,1 and 2. Observing its state at 2 p.m each day, we believe that the system approximately behaves like a homogeneous Markov chain with transition probability matrix

$$P = \begin{bmatrix} 0.6 & 0.2 & 0.2 \\ 0.1 & 0.8 & 0.1 \\ 0.6 & 0.0 & 0.4 \end{bmatrix}$$

Prove that the chain is irreducible, and determine the steady-state probabilities.

17. If $X(t) = Y \cos wt + Z \sin wt$, where Y and Z are two independent normal R.Vs with $E(Y) = E(Z) = 0$, $E(Y^2) = E(Z^2) = \sigma^2$ and w is a constant, prove that $\{X(t)\}$ is a SSS process of order 2.

18. There are 2 white marbles in urn A and 3 red marbles in urn B. At each step of the process, a marble is selected from each urn and the 2 marbles selected are intercharged. Let the state a_i of the system be the number of red marbles in A after i changes. What is the probability that there are 2 red marbles in A after 3 steps ? In the long run, what is the probability that there are 2 red marbles in urn A ?

19. We are given two independent Poisson arrival streams $\{X_t \mid 0 \le f < \infty\}$ and $\{Y_t \mid 0 \le t < \infty\}$ with respective arrival rates λ_n and λ_y. Show that the number of arrivals of the Y_t process occuring between two successive arrivals of X_t process has a modified geometric distribution with parameter ${}^{\lambda x}\!/_{(\lambda x + \lambda y)}$.

20. If $N_1(t)$, $N_2(t)$ are two independent Poisson processes with parameters λ_1, λ_2 respectively, then show that :

$$\Pr\{N_i(t) = k / N_i(t) + N_2(t) = n\} = \left(\hat{k}\right) \rho^k g^{n-k}$$

where $p = \dfrac{\lambda_1}{(\lambda_1 + \lambda_2)}, q = \dfrac{\lambda_2}{(\lambda_1 + \lambda_2)}$.

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AF-2780 **MMA3E3**

M.Sc. DEGREE EXAMINATION, NOVEMBER 2010

Third Semester

Mathematics

Elective—FUZZY MATHEMATICS

(CBCS—2008 onwards)

Time : 3 Hours Maximum : 75 Marks

Part - A $(10 \times 2 = 20)$

Answer **all** questions.

1. Differentiate between a crisp set and a fuzzy set.

2. Define α - cut.

3. Define the union of two fuzzy sets and give an example.

4. Define the complement of a fuzzy set and give an example.

5. What is a cylindric extension ? Explain.

6. Explain : a compatibility relation.

7. Define a fuzzy measure.

8. State Dempster's rule of Combination.

9. What is meant by decision-making in a fuzzy environment ?

10. Give an example of a fuzzy constraint.

Part - B $(5 \times 5 = 25)$

Answer **all** questions.

11. (a) Compute the scalar cardinality and the fuzzy cardinality for the following fuzzy sets :

(i) $A = \cdot 4 / v + \cdot 2 / w + \cdot 5 / x + {}^{\circ}4 / y + 1 / z.$

(ii) $\mu_c (x) = \dfrac{x}{x+1}, x \in \{0, 1, 2, ..., 10\}.$

(*Or*)

(b) Explain the concept of the support of a fuzzy set A in the universal set X. Give an example.

12. (a) State the axioms for fuzzy complement. Explain the Sugeno class of fuzzy Complements.

(*Or*)

(b) Prove that $u\ (a,\ b) = \max\ (a,\ b)$ is the only continuous and idempotent fuzzy set union.

13. (a) Explain max-min composition and max-product composition. If $M_1 = \begin{bmatrix} 1 & 0 & \cdot7 \\ \cdot3 & \cdot2 & 0 \\ 0 & \cdot5 & 1 \end{bmatrix}$ and

$M_2 = \begin{bmatrix} \cdot6 & \cdot6 & 0 \\ 0 & \cdot6 & \cdot1 \\ 0 & \cdot1 & 0 \end{bmatrix}$ find $M_1 \circ M_2$ and $M_1 \odot M_2$.

(*Or*)

(b) Solve the following fuzzy relation equation :

$$p \bullet \begin{bmatrix} \cdot5 & \cdot7 & 0 & \cdot2 \\ \cdot4 & \cdot6 & 1 & 0 \\ \cdot2 & \cdot4 & \cdot5 & \cdot6 \\ 0 & \cdot2 & 0 & \cdot8 \end{bmatrix} = \begin{bmatrix} \cdot5 & \cdot5 & \cdot4 & \cdot2 \end{bmatrix}$$

14. (a) Prove that a belief measure Bel on a finite power set $\mathcal{P}$ (x) is a probability measure if an only if its basic assignment m is given by

$$m(\{x\}) = \text{Bel } (\{x\})$$

and m (A) = 0

for all subsets of X that are not singletons.

(*Or*)

(b) Give a consonant body of evidence $(\mathcal{F}, m)$, prove that the associated Consonant belief and plausibility measures possess the following properties.

(i) $\text{Bel } (A \cap B) = \min (\text{Bel } (A), \text{Bel } (B)]$
for all $A, B \in \mathcal{P}$ (X) ;

(ii) $\text{Pl}(A \cup B) = \max (\text{Pl } (A), \text{Pl } (B)]$
for all $A, B \in \mathcal{P}$ (X).

15. (a) Explain a vector - maximum problem and its optimal compromise solution.

(*Or*)

(b) What is Fuzzy dynamic programming ? Explain.

Part - C (3 × 10 = 30)

Answer any **three** questions.

16. (a) Show that all a - cuts of any fuzzy set A defined on $\mathbb{R}^h$ $(n \geq 1)$ are convex if and only if.

$\mu_A\left[\lambda_r + (1-\lambda)\, s\right] \geq \min\ \left[\mu_A(r),\ \mu_A(s)\right]$ for all r, $s \in \mathbb{R}^h$ and all $\lambda \in [0, 1]$.

(b) Suppose X = {0, 1, 2, ...,10}. Consider the fuzzy sets A, B, C defined on X by

$$\mu_A(x) = \frac{x}{x+2}, \quad \mu_B(x) = 2^{-x},$$

$$\mu_c(x) = \frac{1}{1+10(x-2)^2}.$$

Let $f(x) = x^2$ for all $x \in X$. Use the extension principle to derive $f(A), f(B)$ and $f(C)$.

17. Prove that fuzzy set operations of union, intersection and continuous complement that satisfy the law of excluded middle and the law of contradiction are not idempotent or distributive.

18. (a) Explain the following types of fuzzy relations :

reflexive, symmetric, transitive, asymmetric, anti-transitive.

(b) Explain the algorithm to find the transitive closure of a fuzzy relation. Find the transitive max-min closure for a fuzzy relation defined by the membership matrix.

$$\begin{bmatrix} \cdot7 & \cdot5 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & \cdot4 & 0 & 0 \\ 0 & 0 & \cdot8 & 0 \end{bmatrix}$$

19. Let X = $\{a, b, c, d, e, f, g\}$ and $y = N_7$. Using a joint probability distribution on X × Y given in terms of the matrix

$$\begin{array}{c} \\ a \\ b \\ c \\ d \\ e \\ f \\ g \end{array} \begin{array}{c} \begin{array}{ccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 \end{array} \\ \begin{bmatrix} \cdot08 & 0 & \cdot02 & 0 & 0 & \cdot01 & 0 \\ 0 & \cdot05 & 0 & 0 & \cdot05 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \cdot03 \\ \cdot03 & 0 & 0 & \cdot3 & 0 & 0 & 0 \\ 0 & 0 & \cdot01 & \cdot01 & \cdot2 & \cdot03 & 0 \\ 0 & \cdot05 & 0 & 0 & 0 & \cdot1 & 0 \\ 0 & 0 & 0 & \cdot02 & 0 & \cdot01 & 0 \end{bmatrix} \end{array}$$

Determine :

(i) Marginal probabilities.

(ii) Both conditional probabilities.

(iii) Hypothetical joint probability distribution.

based on the assumption of non-interaction.

20. Consider the following problem :

Minimise $Z = 4x_1 + 5x_2 + 2x_3$

subject to

$$\begin{aligned} 3x_1 + 2x_2 + 2x_3 &\leq 60 \\ 3x_1 + x_2 + x_3 &\leq 30 \\ 2x_2 + x_3 &\geq 10 \\ x_1, x_2, x_3 &\geq 0. \end{aligned}$$

Determine the optimal solution.

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AF-2781 **MMA4C1**

M.Sc. DEGREE EXAMINATION, NOVEMBER 2010

Fourth Semester

Mathematics

FUNCTIONAL ANALYSIS

(CBCS—2008 onwards)

Time : 3 Hours Maximum : 75 Marks

Part - A $(10 \times 2 = 20)$

Answer **all** the questions.

1. State Jensen's inequality.

2. Given an example for a discontinuous linear map.

3. Let X be a normed space over K and f be a non-zeor linear functional on X. If E is an open subset of X, then prove that $f(E)$ is an open subset of K.

4. State Hahn – Banach extension theorem.

5. Write the geometrical interpretation of uniform boundednesss principle.

6. Define the spectrum of a Bounded operator.

7. Define the term "dual basis".

8. Give an example that Norm Covergence implies weak convagence.

9. State and prove polarization identity.

10. Define orthogonal projection.

Part - B $(5 \times 5 = 25)$

Answer **all** the questions.

11. (a) Let Y be a subspace of a normed space X. Prove that Y and its closure $\overline{Y}$ are normed spaces with the induced norm.

(*Or*)

(b) If $F \in BL(X, Y)$, $F \neq 0$ and $\alpha \geq 0$, then prove that $\inf \{\|x\| = 0, x \in X, \|F(x)\| = \alpha\} = \dfrac{\alpha}{\|F\|}$.

12. (a) Let X be a normed space over K, $F \in X'$ and $f \neq 0$. Let $a \in X$ with $f(a) = 1$ and $r > 0$. Then prove that $U(a, r) \, n \, z(f) = \phi$ if and only if $\|f\| \leq \dfrac{1}{r}$.

(*Or*)

(b) Prove that the dual X' of every normed space X is a Banach space.

13. (a) Let X be a normed space and $F: X \to K$ be linear. Prove that *f* is closed if and only if *f* is continuous.

(*Or*)

(b) Prove that $\sigma(A)$ is non-empty.

14. (a) Let X be a normed space. Prove that if X' is separable, then so is X.

(*Or*)

(b) Let $1 \leq p \leq \infty$ and $X = l^p$. Prove that $e_n \not\to 0$ in X, and $e_n \xrightarrow{w} 0$ in X if and only if $1 < p \leq \infty$.

15. (a) State and prove Schwarz inequality.

(*Or*)

(b) State and prove Riesz–Fischer theorem.

Part - C (3 × 10 = 30)

Answer any **three** questions.

16. Let X be a normed space. Prove that the following conditions are equivalent.

(a) Every closed and bounded subset of X is compact.

(b) The subset $\{x \in X : \|x\| \le 1\}$ of X is compact.

(c) X is finite dimensional.

17. Prove that a normed space X is a Banach Space if and only if every absolutely summable series of elements in X is summable in X.

18. State and prove Open mapping theorem.

19. Let X and Y be Banach spaces and $F \in BL(X, Y)$. Prove that R(F) = Y if and only if F' is bounded below.

20. Let H be a Hilbert space, G be a subspace of H and g be a continuous linear functional on G. Prove that there is a unique continuous linear funcional f on H such that $F_{1G} = g$ and $\|f\| = \|g\|$.

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AF-2782 **MMA4C2**

M.Sc. DEGREE EXAMINATION, NOVEMBER 2010

Fourth Semester

Mathematics

NUMBER THEORY

(CBCS—2008 onwards)

Time : 3 Hours Maximum : 75 Marks

Part - A (10 × 2 = 20)

Answer **all** the Questions.

1. If $(a, b) = 1$ and if $c \mid a$ and $d \mid b$, prove that $(c, d) = 1$.

2. If $(a, b) = 1$, prove that $(a + b, a - b)$ is either 1 or 2.

3. Define Möbius function μ and write down the values of $\mu(n)$ for $1 \leq n \leq 10$.

4. Define Liouville's function λ and find the value of $\lambda(12)$.

5. If $x \geq 2$, prove that :

$$\sum_{n \leq x} \frac{1}{\phi(n)} = O(\log x).$$

6. Prove that :

$$(-x) = \begin{cases} -(x) & \text{if } x = (x) \\ -(x) - 1 & \text{if } x \neq (x) \end{cases}$$

7. If $a \equiv b \pmod{m}$, $a \equiv b \pmod{n}$ and $(m, n = 1)$, prove that $a \equiv b \pmod{mn}$.

8. Find out whether the congruence $2x \equiv 3 \pmod{4}$ has an integral solution or not. Justify your answer.

9. What are the quadratic residues and non-residues mod 11 ?

10. Write down the values of $\left(\frac{-1}{17}\right)$ and $\left(\frac{-1}{19}\right)$.

Part - B (5 × 5 = 25)

Answer **all** the Questions.

11. (a) Prove that there are infinitely many primes.

(*Or*)

(b) State and prove the division algorithm.

12. (a) State and prove Möbius inversion formula.

(*Or*)

(b) If $n \geq 1$, prove that :

$$\log n = \sum_{d/n} \wedge(d)$$

13. (a) For $x > 1$, prove that :

$$\sum_{n \le x} \phi(n) = \frac{3}{\pi^2} x^2 + 0(x \log x).$$

(*Or*)

(b) For $x \ge 1$, prove that :

$$\sum_{n \le x} \mu(n) \left[\frac{x}{n}\right] = 1 \text{ and}$$

$$\sum_{n \le x} \wedge(n) \left[\frac{x}{n}\right] = \log(x)!$$

14. (a) Assume $(a, m) = 1$. Prove that :

$$a^{\phi(m)} \equiv 1 \pmod{m}$$

(*Or*)

(b) Find all x which simultaneously satisfy the system of congruences

$$x \equiv 1 \pmod 3, x \equiv 2 \pmod 4, x \equiv 3 \pmod 5)$$

15. (a) Let p be an odd prime. For all n, prove that :

$$\left(\frac{n}{p}\right) \equiv n^{\frac{p-1}{2}} \pmod{p}$$

(*Or*)

(b) Determine whether 888 is a quadratic residue or non-residue of the prime 1999.

Part - C $(3 \times 10 = 30)$

Answer any **three** Questions.

16. (a) If a prime p divides ab, prove that p/a or p/b.

(b) State and prove the fundamental theorem of arithmetic.

17. (a) If f and g are multiplicative functions, prove that their Dirichlet product $f * g$ is multiplicative.

(b) If g is a multiplicative function, prove that its Dirichlet inverse g^{-1} is multiplicative.

18. For all $x \geq 1$, prove that

$$\sum_{n \leq x} d(n) = x \log x + (2C - 1)x + 0\left(\sqrt{x}\right)$$

where C is Euler's constant.

19. State and prove Lagrange's theorem for polynomial congruences.

20. (a) For every odd prime p, prove that :

$$\left(\frac{2}{p}\right) = (-1)^{\frac{p^2-1}{8}} = \begin{cases} 1 & \text{if } \; p \equiv \pm 1 \pmod{8} \\ -1 & \text{if } \; p \equiv \pm 3 \pmod{8} \end{cases}$$

(b) Determine those odd primes p for which 3 is a quadratic residue and those for which it is a non-residue.

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AF-2783 | **MMA4C3**

M.Sc. DEGREE EXAMINATION, NOVEMBER 2010

Fourth Semester

Mathematics

ADVANCED STATISTICS

(CBCS—2008 onwards)

Time : 3 Hours | Maximum : 75 Marks

Part - A $(10 \times 2 = 20)$

Answer **all** Questions.

1. Define (i) Statistical hypothesis and (ii) Likelihood function.

2. Show that, by an example, unbiasedness is not a general property of maximum likelihood estimator.

3. State the properties of sufficient statistic.

4. Explain complex sufficient statistic.

5. Write a note on Bayes confidence interval.

6. Bring out the relationship between sufficient statistic and maximum likelihood estimator.

7. Explain uniformly most powerful term.

8. What is the importance of Likelihood Ratio test ?

9. What is analysis of variance ? What purpose does this technique serve ?

10. Explain the relationship between Likelihood ratio and Correlation coefficient.

Part - B $(5 \times 5 = 25)$

Answer **all** Questions.

11 (a) The observed value of the mean $\bar{X}$ of a random sample of size 20 from a normal distribution $n(\mu, 80)$ is 81.2 Find 90% confidence interval for μ.

(*Or*)

(b) Two independent random samples each of size 10, from two independent normal distributions $n(\mu_1, \sigma^2)$ and $n(\mu_2, \sigma^2)$ yield $\overline{x} = 4.8$, $s_1^2 = 8.64, \overline{y} = 5.6, s_2^2 = 7.88.$ Construct 95% confidence interval for $\mu_1 - \mu_2$.

12. (a) Let $x_1, x_2, \ldots, x_n$ be a random sample of size n from a geometric distribution with p.d.f.

$$f(x;\theta) = (1-\theta)^x \theta, x = 0, 1, 2, \ldots, 0 < \theta < 1,$$
$$= 0 \text{ elsewhere,}$$

Show that $\sum_{i=1}^{n} X_i$ is a sufficient statistic for θ.

(*Or*)

(b) Prove that the n^{th} order statistic of a random sample of size n from a uniform distribution

with p.d.f. $f(x;\theta) = \frac{1}{\theta}, 0 < x < \theta, 0 < \theta < \infty$

$= 0$ elsewhere

is a sufficient statistic for θ.

13 (a) Let $x_1, x_2, \ldots, x_n$ be a random sample of size n from a Poisson distribution that has the mean $\theta > 0$. Prove that the sample mean $\bar{X}$ is an efficient estimator of θ.

(*Or*)

(b) Given the p.d.f. of Cauchy distribution,

$$f(x\ ;\ \theta) = \frac{1}{\pi[1 + (x - \theta)^2]}, -\infty < x < \infty, -\infty < \theta < \infty .$$

Show that the Cramer- Rao lower bound is $\frac{2}{n}$, where n is the sample size.

14 (a) Let $X_1, X_2, \ldots, X_{25}$ be a random sample of size 25 from a normal distribution $n(\theta, 100)$. Find a uniformly most powerful critical region of size $\alpha = 0.10$ for testing $H_0 : \theta = 75$ against $H_1 : \theta > 75$.

(*Or*)

(b) Let X have a p.d.f. $f(x;\theta) = \theta^x (1-\theta)^{1-x}, x = 0,1,$
$= 0$ elsewhere.

Obtain a sequential probability ratio test for testing $H_0 : \theta = \frac{1}{3}$ against $H_1 : \theta = \frac{2}{3}$ for suitable choices of α and β.

15 (a) State and prove Boole's inequality.

(*Or*)

(b) Define a non-central X^2 and derive its p.d.f.

Part - C (3 × 10 = 30)

Answer any **three** Questions.

16. Explain the test for (i) goodness of fit, and (ii) independence of attributes rising Chi-square test.

17. State and prove the Fisher -Neyman theorem for the existence of a sufficient statistic for a parameter.

18. State and establish the Cramer -Rao inequality.

19. Explain the procedure of likelihood ratio test and state its properties.

20. Describe the analysis of variance for two-way classification.

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AF-2784 **MMA4C4**

M.Sc. DEGREE EXAMINATION, NOVEMBER 2010

Fourth Semester

Mathematics

DISCRETE MATHEMATICS

(CBCS—2008 onwards)

Time : 3 Hours Maximum : 75 Marks

Part - A $(10 \times 2 = 20)$

Answer **all** the questions.

1. If A is a set with n elements, how many binary operations can be defined on A ? Prove your answer.

2. Define a sub-semi-group.

3. Let p and q be a two given statements. Write the truth table for the statement $p \leftrightarrow q$.

4. Verify whether the statement $(p \to q) \leftrightarrow (\neg p \vee q)$ is a tautology or not.

5. Determine whether the statement $(\forall x)(p(x)) \to (\exists x)(\mathrm{P}(x))$ is logically valid or not.

6. Verify the validity of the following statements :

   (i) Lions are dangerous animals.

   (ii) There are lions.

   (iii) Therefore there are dangerous animals.

7. What is meant by Hasse diagram ? Give an example.

8. Prove that every chain is a lattice.

9. Define a Boolean algebra.

10. Let L be a Boolean algebra prove that

$a \; v \; (a' \wedge b) = a \vee b$ for all $a, b \in \mathrm{L}$.

Part - B (5 × 5 = 25)

Answer **all** the questions.

11. (a) Prove that a monoid homomorphism preserves the property of invariability.

Or

(b) For any commutative monoid (M,*), prove that the set of idempotent elements of M forms a sub-monoid.

12. (a) Draw the parsing tree for the following :

$$\left(((P \wedge \neg Q) \vee R) \rightarrow (P \vee R)\right)$$

Or

(b) Construct the truth table of the formula

$$\left(P \wedge Q) \vee (\neg P \wedge Q) \vee (P \wedge \neg Q) \vee (\neg P \wedge \neg Q\right)$$

13. (a) Show that $R \to S$ can be derived from the premises

$P \to (Q \to S), \neg R \vee P$ and Q

Or

(b) In the universe of all integers, let Q (x, y) : $x + y = 10$. Examine whether the statement $(\exists x)\ (\forall y)\ Q\ (x, y)$ is true in this universe.

14. (a) Let $(L, \leq)$ be a lattice. Prove that $(a \wedge b) \wedge c = a \wedge (b \wedge c)$ for all $a, b, c, \in L$.

Or

(b) Is the lattice of sub-groups of A_4 a modular lattice ? Prove your contention.

15. (a) Let B be a finite Boolean algebra and $b \neq 0$ in B. Let $a_1, a_2, ..., a_k$ be all the atoms of B such that $a_i \leq b$ then prove that $b = a, \mathrm{V} a_2 \mathrm{~V} ... \mathrm{Va}_k$.

Or

(b) Construct the switching circuit for

$$P = (x_1{}^1\, x_2)^1 + x_3.$$

Part - C (3 × 10 = 30)

Answer any **three** questions.

16. (a) Let S = N × N, N being the set of positive integers and * be an operation on S given by

$(a, b) * (c, d) = (a + c, b + d)$. Show that S is a semi group. Define : $f : (S, *) \to (z, t)$ by $f(a, b) = a - b$. Prove that f is a homomorphism.

(b) If g is a semi group isomorphism from $(S, *)$ to (T,D), prove that g–1 is an isomorphism from (T,D) to $(S, *)$

17. (a) Obtain a disjunctive normal form of

$$P \rightarrow ((P \rightarrow Q) \wedge \neg(\neg Q \vee \neg P)).$$

(b) Find the conjunctive normal form of

$$(q \vee (p \wedge r)) \wedge \neg((p \vee r) \wedge q)$$

18. (a) Derive $P \rightarrow (Q \rightarrow S)$, using the rule CP if necessary, from $P \rightarrow (Q \rightarrow R)$, $Q \rightarrow (R \rightarrow S)$.

(b) Using indirect method of proof, derive $P \rightarrow \neg S$ from $P \rightarrow Q \vee R$, $Q \rightarrow \neg P$, $S \rightarrow \neg R$, P.

19. (a) If G is a group, prove that the set of all normal subgroups of G forms a modular lattice.

(b) Show that the direct product of any two distributive lattices is a distributive lattice.

20. Let B be a finite Boolean algebra and let A be the set of all atoms of B. Prove that the Boolean algebra B is isomorphic to the Boolean algebra P (A).

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AF-2785

MMA4E1

M.Sc. DEGREE EXAMINATION, NOVEMBER 2010

Fourth Semester

Mathematics

Elective—AUTOMATA THEORY

(CBCS—2008 onwards)

Time : 3 Hours Maximum : 75 Marks

Part - A $(10 \times 2 = 20)$

Answer **all** Questions.

1. Describe a model of a discrete automation.

2. Define a non-deterministic finite automation.

3. Define a phrase-structure grammar.

4. Define the languages generalised by a grammar.

5. Consider the grammar G given by

$S \rightarrow O S A_1^2,$

$S \rightarrow 0\,1\,2,$

$2A1 \rightarrow A_1\,2,$

$1A_1 \rightarrow 1\,1.$

Examine whether $0\,0\,1\,1\,2 \in L(G)$. Justify our answer.

6. What is meant by 'Concatenation' ? Explain Illustrate with an example.

7. Describe the set {1, 11, 111, . . .} by regular expression.

8. Find the regular expression to describe the set L = the set of all strings of 0's and 1's ending in 00.

9. Define a derivation tree.

10. Consider G whose production are

$S \rightarrow G\,A\,S \mid a$

$A \rightarrow S\,b\,A \mid S\,S \mid ba$

Show that :

$S \overset{*}{\Rightarrow} a\,a\,b\,b\,a\,a$ and construct a derivation tree whose yield is $a\,a\,b\;\;b\,a\,a$.

Part - B $(5 \times 5 = 25)$

Answer **all** Questions.

11 (a) Prove that for any transition function δ and for any two input strings x and y,

$$\delta(q, xy) = \delta(\delta(q, x), y)$$

(*Or*)

(b) Suppose Q = $\{q_0, q_1, q_2, q_3\}$, $\Sigma = \{0, 1\}$ and F = $\{q^0\}$. Consider the finite state machine whose transition function δ is given in the following table :

	Inputs	
States	0	1
q_0	q_2	q_1
q_1	q_3	q_0
q_2	q_0	q_3
q_3	q_1	q_2

Give the entire sequence of states for the output string 1 1 0 0 0 1.

12. (a) If G = $(\{s\}, \{0, 1\}, \{ s \rightarrow \text{os } 1, s \rightarrow \wedge\} s)$, find L (G).

(*Or*)

(b) Let L be the set of all palindromes over $\{a, b\}$. Construct G generating L.

13 (a) Prove that there exists a recursive set which is not a context-sensitive language over $\{0,1\}$.

(*Or*)

(b) Let $\mathscr{L}_{csl}$ denote the family of contact sensitive languages. Prove that the class $\mathscr{L}_{csl}$ is closed under union.

14 (a) Give an outline of the proof of Arden's theorem : Let P and Q be two regular expressions over Σ. If P does not contain $\wedge$, then the following equation in R, namely,

$$R = Q + RT$$

has a unique solution given by

$$R = QP^*$$

(*Or*)

(b) If L is regular, prove that L^T is also regular.

15 (a) If $A \overset{*}{\Rightarrow} w$ in G, prove that there is a left most derivation of w.

(*Or*)

(b) Let $G = (V_N, \Sigma, P, S)$ be given by the productions :

$S \rightarrow AB,$

$A \rightarrow a,$

$B \rightarrow b,$

$B \rightarrow C,$

$E \rightarrow C.$

Construct V_N', P' and G' such that every variable in G' derives some terminal string.

Part - C (3 × 10 = 30)

Answer any **three** Questions.

16. Give an outline of the proof of the following theorem : For every NDFA, there exists a DFA which simulates the behaviour of NDFA. Alternatively, if L is the set accepted by NDFA, then there exists a DFA which also accepts L.

17. Let G be a type 0 grammar. Prove that we can find an equivalent grammar G_1 in which each production is either of the firm $\alpha \to \beta$, where α and β are strings of variables only, or of the form $A \to a$, where A is a variable and a is a terminal. Further G_1 is of type 1, type 2 or type 3 according as G is of type 1, type 2 or type 3.

18. Prove that a context- sensitive language is recursive.

19. Give an outline of the proof for the following theorem : Every regular expression R can be recognise by a transition system, i.e., for every string w in the set R, there exists a path from the initial state to a final state with path value w.

20. Prove the theorem on the reduction to Chomsky normal form : For every context-free grammar, there is an equivalent grammar G_2 in Chomsky normal form.

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AF-2786 **MMA4E2**

M.Sc. DEGREE EXAMINATION, NOVEMBER 2010

Fourth Semester

Mathematics

Elective — DATA STRUCTURES AND ALGORITHMS

(CBCS—2008 onwards)

Time : 3 Hours Maximum : 60 Marks

Part - A $(10 \times 1\frac{1}{2} = 15)$

Answer **all** the questions

1. Describe program testing, test data and test set.

2. What are the components of time complexity?

3. What is a data object? Give examples.

4. Define a Queue. Define its back and front.

5. Define a Binary tree and state the essential difference between a binary tree and a tree.

6. Define B-trees of order m and state their properties.

7. Describe 0/1 Knapsack problem.

8. Explain the quick sort method.

9. What is the aim of dynamic programming? State the principle of optimality.

10. What are the steps used in a dynamic programming solution?

Part - B (5 × 3 = 15)

Answer **all** questions.

11. (a) What are the techniques used for test data development? Explain briefly.

Or

(b) Explain the various components of time complexity of a program.

12. (a) Explain the concept of Singly Linked List and Chain with neat diagram.

Or

(b) What is stack? Write down the procedure for implementing stock and its operations.

13. (a) Describe the problem of placement of Signal Boosters.

Or

(b) Define a Priority queue. Explain Insert and Delete operations on priority queue.

14. (a) Prove that the greedy algorithm generates optimal loadings.

Or

(b) Develop a non recursive code to locate the min and max in a [0: $n - 1$]

15. (a) Find the solution of [0/1] Knapsack problem using Dynamic Programming.

Or

(b) Describe Backtracking method to search for the solution to a problem.

Part - C $(3 \times 10 = 30)$

Answer any **three** questions.

16. Describe asymptotic notation $\left(\mathrm{O}, \Omega, \theta, o\right)$ and its uses in the performance analysis of programs.

17. Explain the problem 'Towers of Hanoi' and obtain a solution in C++ by means of stacks.

18. Define traversal of a binary tree. Write a C++ program to each type of traversal.

19. Prove that the average complexity of a quick sort is $\theta(n \log n)$ where n is the number of elements to be sorted. Compare various sort methods.

20. State and solve the problem board permutation by means of Branch and Bound algorithm.

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AF-2787 **MMA4E3**

M.Sc. DEGREE EXAMINATION, NOVEMBER 2010

Fourth Semester

Mathematics

Elective — TOPOLOGY-II

(CBCS—2008 onwards)

Time : 3 Hours Maximum : 75 Marks

Part - A $(10 \times 2 = 20)$

Answer **all** the questions.

1. Prove that the Real line R is locally compact.

2. Prove that the one-point compactification of R^2 is homeomorphic to the sphere S^2.

3. Define a completely regular space and give an example.

4. Prove that a completely regular space is regular.

5. Define locally finite collection and countably locally finite collection of subsets of a topological space.

6. Define G_δ and F_σ sets and give examples.

7. Define a complete metric space and give an example.

8. Define a totally bounded metric space and give an example.

9. Prove that the subspace $C(\mathrm{I}, \mathrm{R})$ of continuous functions is not closed in R^{I} in the topology of point wise convergence.

10. Define Baire Space and give an example.

Part - B (5 × 5 = 25)

Answer **all** the questions.

11. (a) Let X be a locally compact space and $f : X \to Y$ be a continuous function. Is the space $f(X)$ locally compact? Justify.

Or

(b) Define one-point compactification Y of a locally compact Hausdorff space X and prove it is a topology on Y.

12. (a) Prove that a product of completely regular spaces is completely regular.

Or

(b) Show that every locally compact Hausdorff space is completely regular.

13. (a) Le a be a locally finite collection of subsets X. Then prove that

(i) Any subcollection of a is locally finite.

(ii) The collection $\beta = \{\overline{A}\}_{A \in a}$ of closures of the elements of a is locally finite.

(iii) $\overline{\bigcup_{A \in a} A} = \bigcup_{A \in a} \overline{A}$

Or

(b) Check whether the following collections of subsets of X are locally finite.

(i) $a = \{(n, n+2) \mid n \in Z\}$

(ii) $\beta = \{(n, 2n) \mid n \in Z_+\}$

(iii) $\mathscr{C} = \{\overline{A} \mid A \in a \text{ which is not locally finite}\}$

14. (a) If (Y, d) is a metric space which is complete, then prove that the space Y^x is complete in the uniform metric $\overline{\rho}$ corresponding to d.

Or

(b) Prove that a sequence f_n of functions converges to the function f in the topology of point-wise convergence if and only if for each $x \in X$, the sequence $f_n(x)$ of points of Y converges to the point $f(x)$.

15. (a) Define Evaluation map and prove it is continuous.

Or

(b) If X is a compact Hausdorff space, then prove that X is a Baire space.

Part - C (3 × 10 = 30)

Attempt any **three** questions.

16. State and prove the Tychnoff Theorem.

17. Prove that there exists a unique StoNe-Cech compactification for a given completely regular space and establish the extension property.

18. If X is a regular space which has a countablity locally finite basis β, then prove that X is normal and every closed set in X is a G_δ set in X.

19. State and prove Ascoli's Theorem.

20. Define compact-open topology and the topology of compact convergence and prove that both topologies coincide on the space $C(X,Y)$ where X is a space and (Y, d) is a metric space.

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