AF-1909 **MMA4E3**

M.Sc. DEGREE EXAMINATION, APRIL 2010

Fourth Semester

Mathematics

Elective - TOPOLOGY - II

(CBCS—2008 Onwards)

Duration : 3 Hours Maximum : 75 Marks

Part - A $(10 \times 2 = 20)$

Answer **all** the Questions

1. Define a locally compact space. Give an example.

2. Define one point compactification of a locally compact Hausdorff Space.

3. Define a completely regular space. Give an example.

4. When do you say two compactifications of a compact Hausdorff space are equivalent ? Give an example.

5. In a metric space prove that every closed set is a $G\delta$ - set.

6. Define Open refinement of a collection of subsets of a space X. give an example.

7. Define a complete metric space and give an example.

8. Define a totally bounded metric space and give an example.

9. Is the collection $\{f_n\}$, where $f_n(x) = x + \sin x$, of subset of $C(R, R)$ pointwise bounded ? Justify.

10. Is a open subset of a Baire space a Baire space ? Justify.

Part - B $(5 \times 5 = 25)$

Answer **all** the Questions

11 a. Let X be a Hansdorff space. Prove that X is locally compact at x if, and only if, for every neighbourhood U of x, there is a neighbourhood V of x such that $\overline{V}$ is compact and $\overline{V} \subset U$.

(Or)

b. Show that X is a Lindeloff space if and only if, for every collection G of subsets of X satisfying the countable intersection condition, $\bigcap_{A \in G} \overline{A} \neq \phi$.

12 a. Show that every locally compact Hausdorff space is completely regular.

(Or)

b. Let X be completely regular. Prove that X is connected if, and only if $\beta(X)$ is connected where $\beta(X)$ is the Stone-Cech compactification of X.

13 a. Let X be a metrizable space. Prove that X has a basis that is countably locally finite.

(Or)

b. Let G be a locally finite collection of subsets of X.

Then prove the following :-

i) Any subcollection of G is locally finite

ii) The collection $\wp = \{\overline{A}\}_{A \in G}$ of the closures of the elements of G is localy finite.

iii) $\bigcup_{A \in G} \overline{A} = \overline{\bigcup_{A \in G} A}$

14 a. Let (X, d) be a metric space. Prove that there is an isometric embedding of X into a complete metric space.

(Or)

b. Let (X, d) be a metric space. If X is complete and totally bounded, prove that X is compact.

15 a. If X is a compact Hausdorff space or a complete metric space prove that X is a Baire space.

(Or)

b. Let X be a locally compact Hausdorff space. Let C(X, Y) have the compact - open topology. Let Z be an arbitary topological space. Prove that a map F : X into $Z \rightarrow Y$ is continuous if, and only if, the induced map $\hat{F} : Z \rightarrow C(X, Y)$ is continuous where $\hat{F}$ is defined by the rule $(\hat{F}(Z))(x) = F(X, Z)$.

Part - C $(3 \times 10 = 30)$

Answer any **three** Questions

16. State and prove the Tychonoff's theorem.

17. Define Stone-Cech compactification of a space X.

Establish (i) the extension property and (ii) uniqueness of Stone-Cech compactification.

18. Let X be a regular space with a basis $\wp$ that is countably locally finite. Prove that X is metrizable.

19. State and prove Peano space - filling curve theorem.

20. State and prove Ascoli's theorem.

AF-1883 **MMA1C2**

M.Sc. DEGREE EXAMINATION, APRIL 2010

First Semester

Mathematics

ANALYSIS - I

(CBCS—2008 Onwards)

Duration : 3 Hours Maximum : 75 Marks

Section - A $(10 \times 2 = 20)$

Answer **All** questions.

1. Define a metric space and give an example.

2. Define a connected set and give an example.

3. Define a complete metric space and give an example.

4. If the sequence $\{p_n\}$ in a metric space X converges prove that it is bounded.

5. If Σa_n converges absolutely prove that Σa_n converges.

6. Define a power series and give an example.

7. Define a continuous function and give an example.

8. Define discontinuity of second kind and give an example.

9. If f is differentiable at a point prove that it is continuous at that point.

10. Define local minimum.

Section - B $(5 \times 5 = 25)$

Answer **All** questions.

11 a. Prove that a set E is open if, and only if, its complement is closed.

(*Or*)

b. Prove that a subset E of the real line R is connected if and only if, x, y$\in$E and x < z < y implies z$\in$E.

12 a. If X is a metric space prove that every convergent sequence is a Cauchy sequence. Further, if X is a compact metric space and if $\{p_n\}$ is a Cauchy sequence in X prove that $\{p_n\}$ converges to some point of X.

(*Or*)

b. Suppose $a_1 \geq a_2 \geq a_3 \geq .... \geq 0$. Prove that the series $\sum_{n=1}^{\infty} a_n$ converges if, and only if the series $\sum_{k=0}^{\infty} 2^k a_{2k}$ converges.

13 a. State and prove Merten's theorem on the product of two non absolutely convergent series.

(*Or*)

b. Suppose the partial sums A_n of Σa_n form a bounded sequence. $b_0 \geq b_1 \geq b_2 \geq \ldots$ and $\lim_{n \to \infty} bn = 0$. Prove that $\Sigma a_n b_n$ converges.

14 a. Prove that a mapping f of a metric space X into a metric space Y is continuous if, and only if, $f^{-1}(C)$ is closed in X for every closed set C in Y.

(*Or*)

b. If f is a continuous mapping of a metric space X into a metric space Y. If E is a connected subset of X prove that f(E) is connected.

15 a. State and prove intermediate value theorem.

(*Or*)

b. State and prove the Chain rule for differentiation.

Section - C (3 × 10 = 30)

Answer any **Three** questions

16. Define a k-Cell. Prove that every k-Cell is compact.

17. State and prove the root test.

18. State and prove Riemann's theorem on rearrangement of series.

19. a) Let f be a continuous mapping of a metric space X into a metric space Y. Prove that f is uniformly continuous on X.

    b) Prove that the continuous image of a compact space is compact.

20. State and prove Taylor's theorem.

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AF-1884 **MMA1C3**

M.Sc. DEGREE EXAMINATION, APRIL 2010

First Semester

Mathematics

DIFFERENTIAL GEOMETRY

(CBCS—2008 Onwards)

Duration : 3 Hours Maximum : 75 Marks

Part - A $(10 \times 2 = 20)$

Answer **All** questions

1. Find the equation of the osculating plane at a point u of the circular helix r = (a cos u, a sin u, bu).

2. What is the curvature and torsion of the curve $x^2 + y^2 = 25$?

3. Define osculating circle and osculating sphere.

4. Define an involute and show that the involutes of a circular helix are plane curves.

5. Calculate the first fundamental coefficients of the anchor ring r = ((b + a cos u) cos v, (b + a cos u) sin v, a sin u).

6. Find the orthogonal trajectories of the circle r = a cosθ.

7. Show that a curve on a plane is a geodesic if and only if, it is a straight line.

8. Find the geodesic curvature of the curve u = constant on the surface given by r = (u cos v, u sin v, $^1/_2$ au^2).

9. Show that the points of the helicoid r = (u cos v, u sin v, av) are hyperbolic.

10. Define polar developable and rectifying developable of a space curve.

Part - B (5 × 5 = 25)

Answer **All** questions

11 a. Find the curvature and torsion of r = (u, u^2, u^3).

(*Or*)

b. Prove that the length of the common perpendicular d between the tangents at two neighbouring points with the arcual distance s between them is approximately $d = \dfrac{k \tau s^3}{12}$

12 a. Find the involutes and evolutes of the circular helix
r = (a cosθ, a sinθ, bθ).

(*Or*)

b. Find the intrinsic equation of the curve.
r = (a e^u cos u, a e^u sin u, be^u)

13 a. Find the surface of revolution which is isometric with the regioin of the right helicoid.

(*Or*)

b. The metric on the surface is $v^2\, du^2 + u^2\, dv^2$. Find the family of curves orthogonal to the curve uv = constant and find the metric referred to new parameters so that these two families and parametric curves.

14 a. State and prove the normal property of Geodesics.

(*Or*)

b. Obtain the geodesic equations using Christoffel symbols of first kind.

15 a. State and prove Euler's theorem.

(*Or*)

b. Prove that the edge of regression of the rectifying developable has the equation $R = r + \dfrac{k\,(\tau\hat{t} + k\hat{b})}{(k'\tau - k\tau')}$

Part - C (3 × 10 = 30)

Answer any **Three** questions

16. Find the curvature and torsion of the curve of intersection of the quadratic surfaces $ax^2 + by^2 + cz^2 = 1$ and $a'\,x^2 + b'\,y^2 + c'\,z^2 = 1$.

17. State and prove the fundamental existence theorem for space curves.

18. Show that the parametric curves on the sphere given by x = a sin u, cos v, y = a sin u sin v, z = a cos u where $0 < u < \frac{\pi}{2}$, $0 < v < 2\pi$ form an orthogonal system. Determine the two families of curves. which meet the curve v = constant at angles $\frac{\pi}{4}$ and $\frac{3\pi}{4}$. Find the metric of the surface referred to these two families as parametric curves.

19. Obtain a necessary and sufficient condition for a curve u = u(t), v = v(t) on a surface r = r(u,v) to be a geodesic.

20. Prove that a necessary and sufficient condition for a surface to be a developable is that its Gaussian curvature is Zero.

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AF-1885 **MMA1C4**

M.Sc. DEGREE EXAMINATION, APRIL 2010

First Semester

Mathematics

DIFFERENTIAL EQUATIONS

(CBCS—2008 Onwards)

Duration : 3 Hours Maximum : 75 Marks

Part - A $(10 \times 2 = 20)$

Answer **all** Questions

1. Are the solutions e^{2x}, xe^{2x} of $y^{11} - 4y^{1} + 4y = 0$. Linearly independent on any interval ? Justify.

2. If $\phi_1(x) = e^x$ is a solution of $xy^{11} - (x + 1)\, y^1 + y = 0$, find a second independent solution.

3. Find the singular points of the equation.
$(x^2 + x - 2)^2\, y^{11} + 3\,(x + 2)\, y^1 + (x - 1)\, y = 0$.

4. If $J_p(x)$ denotes the Bessel function of order p,
Prove that $\frac{d}{dx}\,(x^p J_p(x)) = x^p J_{p-1}\,(x)$

5. Eliminate constant a and b from $z = (x + a)\,(y + b)$.

6. Find a complete integral of the equation.

   $(p + q)(z - xp - yq) = 1.$

7. If $u = f(x + iy) + g(x - iy)$, where the function f and g are arbitary,

   Prove that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$

8. Find a particular integral of the equation $(D^2 - D^1) z = 2y - x^2$.

9. When r, θ, ϕ are spherical polar coordinates. Prove that $r\cos\theta$ and $r^{-2}\cos\theta$ satisfy Laplace equation.

10. Write down the one dimensional wave equation and d^1 Alembert's solution of the one dimensional wave equation.

Part - B (5 × 5 = 25)

Answer **all** Questions

11 a. One solution of $x^3 y^{111} - 3x^2 y^{11} + 6x y^1 - 6y = 0$ for $x > 0$ is $\phi_1(x) = x$. Find a basis for the solutions for $x > 0$.

(Or)

b. Find the solution ϕ of $(1 + x^2) y^{11} + y = 0$ of the form

$$\phi(x) = \sum_{k=0}^{\infty} C_k x^k$$ which satisfies $\phi(0) = 0, \phi^1(0) = 1.$

12 a. Consider the equation $x^2 y^{11} + x e^x y^1 + y = 0$. Compute the indicial polynomial and show that its roots are i and -i. Compute the coefficients C_1, C_2, C_3 in the solution $\phi(x) = x^i \sum_{k=0}^{\infty} C_k x^k$ (c0 = 1).

(Or)

b. Show that $x^{1/2} J_{1/2}(x) = \dfrac{\sqrt{2}}{\Gamma\left(\frac{1}{2}\right)} \sin x$ and

$x^{1/2} J_{-1/2}(x) = \dfrac{\sqrt{2}}{\Gamma\left(\frac{1}{2}\right)} \cos x$

13 a. Find the general solution of the differential equation

$x^2 \dfrac{\partial z}{\partial x} + y^2 \dfrac{\partial z}{\partial y} = (x + y) z$

(Or)

b. Show that the equations $xp - yq = x$, $x^2p + q = xz$ are compatible and solve them.

14 a. Solve the equation $\dfrac{\partial^3 z}{\partial x^3} - 2 \dfrac{\partial^3 z}{\partial x^2 \partial y} - \dfrac{\partial^3 z}{\partial x \partial y^2} + 2 \dfrac{\partial^3 z}{\partial y^3} = e^{x+y}$.

(Or)

b. Reduce the equation $\dfrac{\partial^2 z}{\partial x^2} + 2 \dfrac{\partial^2 z}{\partial x \partial y} + \dfrac{\partial^2 z}{\partial y^2} = 0$ to canonical form and hence solve it.

15 a. A rigid sphere of radius 'a' is placed in a stream of fluid whose velocity in the undisturbed state is V. Determine the velocity of the fluid at any point of the disturbed stream.

(Or)

b. Described the motion of the string which is released from rest in the position $y = \frac{4\in}{l^2} x (e - x)$

Part - C (3 × 10 = 30)

Answer any **three** Questions

16. Compute two linearly independent series solutions for 1 x 1 < 1, for the equation $(1 - x^2) y^{11} - xy^1 + \alpha^2 y = 0$, where α is a constant. Prove that for every non-negative integer $\alpha = n$, there is a polynomial solution of degree n.

17. Let $\alpha > 0$ and λ, k be positive zeroes of J_α. If $\alpha \neq \mu$ Prove that $\int_0^1 \phi_\lambda(x)\, \phi_\mu(x)\, dx = \int_0^1 x\, J\alpha\, (\lambda x).\, J_\alpha\, (\mu x)\, dx = 0.$

and if $\lambda = \mu$ Prove that $\int_0^1 \phi^2_\lambda\, (x)\, dx = \int_0^1 x\, J_\alpha^{\,2}\, (\lambda x)\, dx = \frac{1}{2} (J_\alpha^{\,1}\, (\lambda))^2.$

18. Find the complete integral of the equation $px^5 - 4q^3 x^2 + 6x^2 z - 2 = 0$ using Jacobi's method.

19. By separating the variables, show that the equation $\nabla_1^2 V = 0$ has solutions of the form $A \exp(\pm n x \pm iny)$, where A and n are constants. Deduce that functions of the form

$$V(x, y) = \sum_r Ar\, e^{\frac{-r\pi x}{a}} \operatorname{Sin} \frac{r\pi y}{a} x \geq 0,\ 0 \leq y \leq a$$

Where the Ar's are constant, are plane harmonic functions satisfying the conditions $v(x, 0) = 0$, $v(x, a) = 0$, $v(x, y) \to 0$ as $x \to \infty$.

20. Prove that the total energy of a string which is fixed at the points $x = 0$, $x = l$ and is executing small transverse vibrations is

$$w = \frac{1}{2} T \int_0^l \left\{ \left(\frac{\partial y}{\partial x}\right)^2 + \frac{1}{C^2} \left(\frac{\partial y}{\partial t}\right)^2 \right\} dx.$$

Show that if $y = f(x - ct)$, $0 \leq x \leq l$, then the energy of the wave is equally divided between potential energy and kinetic energy.

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AF-1886 **MMA 1E1**

M.Sc DEGREE EXAMINATION, APRIL 2010

I Semester

MATHEMATICS

ELECTIVE - MECAHNICS

(CBCS - 2008 Onwards)

Duration : 3 Hours Maximum : 75 marks

Part - A (10 x 2 = 20)

Answer ALL Questions

1. Define angular momentum and the moment of a force about a point.

2. Give examples of rheonomous and scleronomous constraints.

3. Find the motion of one particle using Cartesian coordinates.

4. Write the Lagrangian for a charged particle in an electromagnetic field.

5. What are monogenic systems?

6. If the translation coordinate q_j is cyclic, what is the generalized force Q_j?

7. What is the condition for the orbit to be a circle, under inverse square law of force?

8. Show that the elliptic orbit is invariant under reflection about the apsidal vectors.

9. State Kepler's first law.

10. What is the direction and magnitude of the Laplace-Runge-Lenz vector $\overline{A}$?

Part-B (5 x 5 = 25)

Answer ALL Questions

11. a. If the forces acting on a particle are conservative, prove that the total energy on the particle, T + V is conserved.

(OR)

b. Discuss the motion of a single particle using plane polar coordinates.

12. a. Obtain Lagrange's equation of motion for Raleigh's dissipation function.

(OR)

b. Give an example where Lagrange's equation of motion is applied for a conservative system with holonomic constraint.

13. a. Show that geodesics of a spherical surface are great circles.

(OR)

b. Show that central force motion of two bodies about their centre of mass can always be reduced to an equivalent one body problem.

14. a. With usual notation, prove that

$$\frac{1}{2}m\dot{r}^2 + \frac{1}{2}\frac{\ell^2}{mr^2} + V = \text{constant.}$$

(OR)

b. State and prove Virial theorem.

15. a. For the inverse square law of forces, prove that the orbit is a conic with eccentricity $e = \sqrt{1 + \frac{2E\ell^2}{mk^2}}$.

(OR)

b. Using Kepler's equation $wt = \psi - e\sin\psi$, prove that

$$\tan\frac{\theta}{2} = \left(\frac{1+e}{1-e}\right)^{1/2} \tan\frac{\psi}{2}.$$

Part-C (3 x 10 = 30)

Answer any THREE Questions

16. State D'Alembert's principle and hence obtain Lagrange's equation of motion for a holonomic system.

17. A particle moves in a plane under the influence of a force, acting toward a centre of force whose magnitude is

$$F = \frac{1}{r^2}\left[1 - \frac{\dot{r}^2 - 2\ddot{r}r}{c^2}\right],$$ where r is the distance of the particle to the centre of force. Find the generalized potential that will result in such a force and from that the Lagrangian for the motion in a plane.

18. Derive Euler-Lagrange differential equations.

19. Show that the central force problem is solved in terms of elliptic functions when the force is a power law function if the distance with the integral exponents $n = +5, +3, 0, -4, -5, -7$.

20. For circular and parabolic orbits in an attractive $\frac{1}{r}$ potential having the same angular momentum, show that perihelion distance of the parabola is one half of the radius of the circle.

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AF-1887 MMA1E2

M.Sc. DEGREE EXAMINATION, APRIL 2010

First Semester

Mathematics

ELECTIVE - PROGRAMMING IN C++

(CBCS—2008 Onwards)

Duration : 3 Hours Maximum : 60 Marks

Part - A $(10 \times 1\frac{1}{2} = 15)$

Answer **All** Questions.

1. Distinguish between Data abstraction and Data encapsulation.

2. What is a reference variable ? what is the major use of this variable ?

3. Write down the difference between a structure and a class in C++.

4. Write a function SWAP to exchange the values of two float type variables.

5. Distinguish between the following two statements :

   Time T2 (T2) ;

   Time T2 = T1; Where T1 and T2 and objects of time class.

6. A class ALPHA has a constructor as follows :

   ALPHA (int a, double b) ;

   Can we use this constructor to convert types ? Justify.

7. What is an abstract class ? Give an example.

8. What is container ship ? How does it differ from inheritance ?

9. What is the basic difference between manipulators and ios member functions in implementation ?

10. How many file objects would you need to create to manage the following situations ?

    i) To process four files sequentially.

    ii) To merge two sorted files into a third file.

Part - B (5 × 3 = 15)

Answer **All** Questions.

11 a. What is object oriented programming ? What are its main characteristics ?

(*Or*)

b. Write the general form and explain the functions of the following statements :

    i) do - while

    ii) Switch

12 a. Write a function POWER( ) to raise a number m to a power n. The function takes a double value for 'm' and int value for 'n' and returns the result correctly. Use a default value of 2 for n to make the function to calculate squares when this argument is omitted. Write a main that gets the values of m and n from the user to test the function.

(*Or*)

b. Explain with example

i) arrays of objects and

ii) Objects as function arguments.

13 a. What is a constructor ? Explain with example

i) Multiple Constructors

ii) Parameterized Constructors.

(*Or*)

b. What is operator overloading ? Why is it necesary to overload an operator ? State the rules for overloading operators.

14 a. How will you make a private member of a class inheritable ? Explain with an example.

(*Or*)

b. Explain with examples how the constructors in derived classes work ?

15 a. Explain the functions of

i) put( ) and get( ) functions

ii) getline( ) and write( ) functions.

(*Or*)

b. What is a file mode ? describe the various file mode options available.

Part - C $(3 \times 10 = 30)$

Answer any **Three** questions.

16. Write a program in C++ to solve the quadratic equation $ax^2 + bx + c = 0$

17. Create two classes DB and DM which store the values of distances. DB stores distances in feet and inches where as DM stores distances in metres and centimetres. Write a program that can read values for the class objects and add one object of DB with another object of Dm and stores the result in an object of DM. Use a fried function to carryout the addition operation. Write a main program to test your classes.

18. Explain with suitable examples the three types of conversity between uncompatible types of data.

19. What does inheritance mean in C++ ? What are the different forms of inheritance ? Explain each with an example.

20. A file named DATA contains a series of integer numbers. Code a program to read these numbers and then write all order numbers to a file to be called ODD and all even numbers to a file to be called EVEN.

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AF-1888 **MMA1E3**

M.Sc. DEGREE EXAMINATION, APRIL 2010

First Semester

Mathematics

CALCULUS OF VARIATIONS AND SPECIAL FUNCTIONS

(CBCS—2008 Onwards)

Duration : 3 Hours Maximum : 75 Marks

Part - A $(10 \times 2 = 20)$

Answer **all** Questions

1. Write down the Euler - Lagrange differential equation.

2. Define geodesics with an example.

3. Write the Ostrogradsky equation.

4. State Hamilton's principle.

5. State Rayleigh's principle.

6. Write the transversality condition.

7. What is the expansion of Jo ?

8. What are the Legendre polynomials $p_1(x)$ and $p_2(x)$?

9. The value of $\pi(^1/_2)$ = ...

10. State Rodrigue's formula.

Part - B (5 × 5 = 25)

Answer **all** Questions

11 a. Show that the straight line is the shortest distance between two points in a plane.

(Or)

b. Find the goedesics on sphere of radius a.

12 a. Find the transversality condition for the functional

$$v = \int_{x0}^{x1} A(x, y)\sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

(Or)

b. Find the shortest distance between the point A(-1, 3) and the straight line y = 1 - 3x.

13 a. Explain the Ritz method.

(Or)

b. Find the approximate solution of Poisson's equation

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = -1 \text{ in the rectangle } D = \begin{cases} -a \le x \le a \\ -b \le y \le b \end{cases}$$

14 a. Prove that $\beta(m, n) = \dfrac{\Gamma(m) \cdot \Gamma(n)}{\Gamma(m+n)}$

(Or)

b. Express $j_r(x)$ in terms of $J_0(x)$ and $J_1(x)$

15 a. Define Chebysher polynomials. Write down it's generating function and orthogonal property.

(Or)

b. Prove that $H_n(x) = (-1)^n e^{x^2} \dfrac{dn}{dx^n} (e^{-x^2})$

Part - C (3 × 10 = 30)

Answer any **three** Questions

16. Derive Euler - Lagrange differential equation.

17. Prove that the sphere is the solid figure of revolution, which for a given surface are a has maximum volume.

18. Find an approximate solution of the problem of the minimum of the functional

$v(y(x)) = \int_0^1 (y^{112} - y^2 - 2xy)\, dy;\ y(0) = y(\partial) = 0$

and compare it with the exact solution.

19. Solve the equation $x \dfrac{d^2y}{dx^2} + \dfrac{dy}{dx} + xy = 0$

20. Obtain the generating function of $p_n(x)$

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AF-1889

MMA2C1

M.Sc. DEGREE EXAMINATION, APRIL 2010

Second Semester

Mathematics

ALGEBRA—II

(CBCS—2008 Onwards)

Duration : 3 Hours

Maximum : 75 Marks

Part - A $(10 \times 2 = 20)$

Answer **All** the questions.

1. If S and T are non - empty subsets of a vector space V over F, prove that $L(S \cup T) = L(S) + L(T)$.

2. Are the vectors $(1, 0, 1), (5, -3, -1), (1, -1, -1)$ linearly independent ? Justify.

3. Let V be an inner product space over a field F and let W be a subspace of V. Define the orthogonal complement of W and show that it is a subspace of V.

4. If S, T, $\in$ Hom (V, W) and v_i S = v_i T for all elements v_i of a basis of V, prove that S = T.

5. Exhibit a polynomial of degree 4 over the field of rationals satisfied by $\sqrt{2} + \sqrt{3}$.

6. Find the degree of the splitting field of the polynomial $x^4 + x^2 + 1$ over the field of rational numbers.

7. Let K be the field of complex numbers and let F be the field of real numbers. Find G (K, F).

8. Let K be a field and let F be a subfield of K. Prove that G (K, F) is a subgroup of the group of all automorphisms of K.

9. Prove that the element $\lambda \in F$ is a characteristic root of $T \in A(V)$ if, and only if, $vT = \lambda v$ for some $v \neq 0$ in V.

10. If T is skew - Hermitian, prove that all of its characteristic roots are pure imaginaries.

Part - B (5 × 5 = 25)

Answer **All** questions.

11. (a) If V is finite dimensional over F prove that any two bases of V have the same number of elements.

(*Or*)

(b) If $V_1, V_2, ... V_n$ in V have W as linear span and if $v_1, v_2, ... v_k$ are linearly independent show that we can find a subset of $V_1, V_2, ... V_n$ of the form $V_1, V_2, ... V_k$, $v_{i1}, ... v_{ir}$ consisting of linearly independent elements whose linear span is also W.

12. (a) If V is finite dimensional and W is a subspace of V prove that W is isomorphic to $\frac{\hat{V}}{A(W)}$ and dim A(W) = dim V – dim W.

(*Or*)

(b) State and prove schwarz in equality.

13. (a) Let K be an extension of a field F. Prove that the elements in K which are algebraic over F form a subfield of K.

(*Or*)

(b) If $p(x)$ is a polynomial in F[x] of degree $n \geq 1$ and is irreducible over F prove that there is an extension E of F such that [E : F] = n, in which p (x) has a root.

14. (a) If K is a finite extension of F prove that G (K, F) is a finite group and $\mathrm{O}\left(\mathrm{G}\left(\mathrm{K}, \mathrm{F}\right)\right) \leq \left[\mathrm{K} : \mathrm{F}\right]$.

(*Or*)

(b) Let f (x) be a polynomial in F [x], K its splitting field over F, G (K, F) its Galois group and T any subfield of K containing F. Prove that T is a normal extension of F if, and only if, G (K, T) is a normal subgroup of G (K, F)

15. (a) If V is finite dimensional over F, prove that $T \in A(V)$ is regular if, and only if, T maps V onto V.

(*Or*)

(b). Prove that the linear transformation T on V is unitary if, and only if, it takes an orthonormal basis of V into an orthonormal basis of V.

Part - C (3 × 10 = 30)

Answer any **Three** the questions**.**

16. If V is finite dimensional and if W is a subspace of V prove that W is finite dimensional dim W ≤ dim V and dim $\left(\frac{V}{W}\right) = \dim V - \dim W$.

17. If V and W are vector spaces over F of dimensions *m* and *n* respectively prove that Hom (V, W) is also a vector space over F and is of dimension *m.n*.

18. Let K be an extension of a field F. Prove that $a \in K$ is algebraic over F if, and only if, F (a) is a finite extension of F.

19. Let K be an extension of a field F. Prove that K is a normal extension of F if, and only if, K is the splitting field of some polynomial over F.

20. If $T \in A(V)$ has all its characteristic roots in F prove that there is a basis of V in which the matrix of T is triangular.

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AF-1890 **MMA2C2**

M.Sc. DEGREE EXAMINATION, APRIL 2010

Second Semester

Mathematics

ANALYSIS—II

(CBCS—2008 Onwards)

Duration : 3 Hours Maximum : 75 Marks

Part - A $(10 \times 2 = 20)$

Answer **All** questions.

1. If P* is a refinement of p, prove that $L(P, f, \alpha) \leq L(P^*, f, \alpha)$

2. If $f \in \Re(\alpha)$ on $[a, b]$ and if k is a constant, prove that $k\, f \in \Re(\alpha)$ and $\int_a^b k\, f\, d\alpha = k \int_a^b f\, d\alpha$.

3. Define uniform convergence of sequence of functions.

4. Prove by an example that the limit of the integral is not equal to the integral of the limit.

5. Prove that $\lim_{x \to 0} \dfrac{\log(1+x)}{x} = 1$.

6. Find $\lim_{x \to 0} \frac{x - \sin x}{\tan x - x}$.

7. If E_1 and E_2 are measurable prove that $E_1 \cup E_2$ is also measurable

8. If f is a measurable function and $f = g$ a.e. prove that g is measurable.

9. If f is integrable over E prove that $|f|$ is integrable.

10. If $f(x) = \begin{cases} 0 \text{ if } x \text{ is irrational} \\ 1 \text{ if } x \text{ is rational,} \end{cases}$ find $R \int_a^{-b} f(x)\, dx$ and $R \int_{-a}^{b} f(x)\, dx$.

Part - B (5 × 5 = 25)

Answer **All** questions.

11. (a) Let $f \in \Re(\alpha)$ and $g \in \Re(\alpha)$ on [a, b]. Prove that :

(i) $f + g \in \Re(\alpha)$.

(ii) $f \cdot g \in \Re(\alpha)$ and

(iii) $|f| \in \Re(\alpha)$.

(Or)

(b) Prove that $f \in \Re(\alpha)$ on $[a, b]$ if, and only if, for every $\in > 0$ there exists a partition p such that $U(P, f, \alpha) - L(P, f, \alpha) < \in$.

12. (a) State and prove Cauchy's criterion for uniform convergence.

(*Or*)

(b) If K is compact, $f_n \in \ell(k)$ for $n = 1, 2, 3, \ldots$ and if $\{f_n\}$ is pointwise bounded and equicontinuous on K prove that (i) $\{f_n\}$ is uniformly bounded on K and (ii) $\{f_n\}$ contain a uniformly convergent subsequence.

13. (a) If f is a positive function on $(0, \infty)$ such that (i) $f(x+1) = x f \in$; (ii) $f(1) = 1$; (iii) $\log f$ is convex prove that $f(x) = \Gamma(x)$.

(*Or*)

(b) Suppose the series $\sum a_n x^n$ and $\sum b_n x^n$ converge in the segment $S = (-R, R)$. Let E be the set of all $x \in S$ at which $\sum_{n=0}^{\infty} a_n x^n = \sum_{n=0}^{\infty} b_n x^n$. If E has a limit point in S prove that $a_n = b_n$ for $n = 0, 1, 2, \ldots$ and the above equality holds for all $x \in S$.

14. (a) Let $< E_n >$ be an infinite decreasing sequence of measurable sets with $E_{n+1} \subset E_n$ for each n. Let $m(E_1)$ be finite. Prove that $m\left(\bigcap_{i=1}^{\infty} E_i\right) = \lim_{n \to \infty} mE_n$.

(*Or*)

(b) Does there exist a non-measurable set ? Justify your answer.

15. (a) If f and g are bounded measurable functions defined on a set E of finite measure prove that $\int_E (af + bg) = a\int_E f + b\int_E g$. Further, if A and B are disjoint measurable sets of finite measure, prove that $\int_{A \cup B} f = \int_A f + \int_B f$.

(*Or*)

(b). State and prove Lebesgue convergence theorem.

Part - C (3 × 10 = 30)

Answer any **Three** the questions.

16. Let $f \in \Re$ on [a, b]. For $a \leq x \leq b$, put $F(x) = \int_a^x f(t)\,dt$. Prove that F is continuous on $[a, b]$. Further, if f is continuous at a point x_0 of $[a, b]$ prove that F is differentiable at x_0 and $F'(x_0) = f(x_0)$. Derive the fundamental theorem of calculus

17. State and prove the Stone - Weierstrass theorem.

18. State and prove Parseval's theorem.

19. Prove that the outer measure of an interval is its length.

20. (a) State and prove Bounded convergence theorem.

    (b) State and prove monotone convergence theorem.

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AF-1891	MMA2C3

M.Sc. DEGREE EXAMINATION, APRIL 2010

Second Semester

Mathematics

NUMERICAL ANALYSIS

(CBCS—2008 Onwards)

Duration : 3 Hours — Maximum : 75 Marks

Part - A $(10 \times 2 = 20)$

Answer **all** Questions

1. Find the critical points of $F(x) = x_1^3 + x_2^3 - 2x_1^2 + 3x_2^2 - 8$.

2. Find the spectral radius of the matrix $\begin{pmatrix} 2 & 3 \\ 4 & 1 \end{pmatrix}$

3. Define Orthogonal polynomials. Give Examples.

4. Define Hermite polynomials $H_k(x)$ and find $H_2(x)$ and $H_3(x)$.

5. Find $\frac{dy}{dx}$ at $x = 1.2$ from the following data

x :	1.2	1.3	1.4	1.5	1.6
y :	1.5095	1.6984	1.9043	2.1293	2.3756

6. Write down simpson's $\frac{1}{3}$ - formula and simpson's $\frac{3}{8}$ - formula for for integration.

7. Find the general solution of $y^n - 5y^1 + 6y = 0$.

8. Using Euler's method, find $y(0.1)$ and $y(0.2)$, given that $y^1 = 1 + xy$ with $y(0) = 1$.

9. Write Adams - Bashforth formula to solve a differential Equation.

10. What do you mean by an initial value problem. Give an example.

Part - B (5 × 5 = 25)

Answer **All** questions

11 a. Solve the system

$$\begin{aligned} 4x_1 - x_2 &= 1 \\ 4x_2 - x_1 - x_3 &= 1 \\ 4x_3 - x_2 - x_4 &= 1 \\ 4x_4 - x_3 &= 1 \end{aligned}$$

by Jacobi iteration method

(*Or*)

b. Find the real root of $x e^x - 2 = 0$ correct to three places of decimals using Newton's method.

12 a. Calculate a good polynomial approximation of degree n on $0 \le x \le 1$ to $f(x) = \sqrt{x}$ for n = 1, 2, 3, 4, 5.

(*Or*)

b. Calculate the polynomial of degree ≤ 2 which minimizes
$\int_{-1}^{+1} (\sin \pi x - p(x))^2 \, dx$ over all polynomials p(x) of degree ≤ 2.

13 a. Derive the composite trapezoid rule T_N and the composite midpoint rule M_N.

(*Or*)

b. The function f(x) is defined on [0, 1] as follows :

$$f(x) = x \qquad 0 \leq x \leq {}^1/_2$$
$$= 1 - x \qquad {}^1/_2 \leq x \leq 1.$$

Calculate $\int_0^1 f(x)$ using

(i) Trapezoidal rule

(ii) Simpson's ${}^1/_3$ rule.

14 a. Using Taylor's series method, find the solution of the differential equation $xy^1 = x - y \quad y(2) = 2$ at x = 2.1 correct to 5 decimal places.

(*Or*)

b. If $y_0 = 1$, $y_1 = x$, Show that the nth term $y_n = y_n(x)$ of the solution of $y_{n+2} - 2x\, y_{n+1} + y_n = 0$ is a polynomial of degree n in x with leading coefficient 2^{n-1}.

15 a. Solve the finite difference methods the boundary value problem

$$\frac{d^2y}{dx^2} + y = 0 \quad y(0) = 0, y(1) = 1$$

(*Or*)

b. Solve the non-linear boundary value problem $yy^{11} + 1 + y^{12} = 0$, $y(0) = 1$, $y(1) = 2$ by the shooting method.

Part - C $(3 \times 10 = 30)$

Answer any **Three** questions

16. Solve the system $f(\varepsilon) = 0$ with $f_1(x) = x_1 + 3 \ln |x_1| - x_2^2$
$f_2(x) = 2x_1^2 - x_1 x_2 - 5 x_1 + 1$ using Demped Newton's method.

17. Using the appropriate recurrence relation, generate the first five Hermite polynomials. Are the polynomials orthogonal ? Justify.

18. Using simpson's rule calculate an approximation to the integral $I = \int_0^1 (1 + x^2)^{3/2} dx$ which are correct to 6 decimal places.

19. Using Runge kutta method of order four, calculate the value of y when x = 0.1 and x = 0.2, given that $\frac{dy}{dx} = x + y, \quad y(0) = 1.$

20. Using Milne's predictor corrector method, find y(0.6), (taking h = 0.2), for the equation $y^1 = y - x^2$, $y(0) = 1$, obtaining the starting values by Taylor's series method.

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AF-1892 | **MMA2C4**

M.Sc. DEGREE EXAMINATION, APRIL 2010

Second Semester

Mathematics

PROBABILITY AND STATISTICS

(CBCS—2008 Onwards)

Duration : 3 Hours | Maximum : 75 Marks

Part - A $(10 \times 2 = 20)$

Answer **All** questions

1. Let x have the p.d.f. $f(x) = 2(1 - x), 0 < x < 1$
$= \text{Zero else where.}$
Find $E(x)$ and $E(x^2)$

2. Let $f(x) = \dfrac{x}{6}$, $x = 1, 2, 3$, Zero else where be the p.d.f. of x.
Find the distribution function and the p.d.f. of $y = x^2$.

3. Let the joint p.d.f. of x_1 and x_2 be
$$f(x_1, x_2) = \frac{x_1 + x_2}{21} \quad x_1 = 1, 2, 3, \quad x_2 = 1, 2 \text{ and}$$
$= \text{Zero elsewhere}$. Find the marginal p.d.f.'s.

4. Let the joint p.d.f. of x_1 and x_2 be $f(x_1, x_2) = 12\, x_1\, x_2\, (1 - x_2)$, $0 < x_1 < 1, 0 < x_2 < 1$, Zero elsewhere. Are x_1 and x_2 Stochastically independent ? Justify.

5. X is a binomial variate with parameters $n = 7$ and $p = \frac{1}{2}$

Find its moment generating function and $P(x = 5)$.

6. Find x is n (75, 100), find Pr $(x < 60)$ and Pr $(70 < x < 100)$.

7. Determine the constant c so that $f(x) = cx(3 - x)^4$, $0 < x < 3$. Zero else where, is a p.d.f.

8. Let x have the p.d.f. $f(x) = \frac{x^2}{9}$, $0 < x < 3$, Zero else where.

Find the p.d.f. of $y = x^3$.

9. Let $\overline{x}$ denote the mean of a random sample of size 100 from a distribution which is x^2 (50). Compute an approximate value of $Pr(49 < \overline{x} < 51)$.

10. Let Zn be $x^2(n)$ and let $Wn = \frac{Zn}{n^2}$. Find the limiting distribution of Wn.

Part - B (5 × 5 = 25)

Answer **All** questions

11 a. There are five red chips and three blue chips in a bowl. The red chips are numbered 1, 2, 3, 4, 5 respectively and the blue chips are numbered 1, 2, 3 respectively. If two chips are to be drawn at random and without replacement find the probability that these chips have either the same number or the same colour.

(*Or*)

b. Let $f(x) = 6x(1 - x)$, $0 < x < 1$, Zero else where be the p.d.f. of the random variable x. Find the moment generating function, the mean and the variance of x.

12 a. Let the random variables x and y have the joint

p.d.f. $\quad f(x, y) = x + y, \; 0 < x < 1, \; 0 < y < 1$

$\qquad\qquad = 0$ else where

Find the correlation coefficient between x and y.

(*Or*)

b. Let $f(x_1, x_2) = e^{-x_1 x_2}$, $0 < x_1 < \infty$, $0 < x_2 < \infty$, zero else where be the joint p.d.f of the random variables x_1 and x_2. Show that x_1 and x_2 are stochastically independent and that

$E(e^t(x_1 + x_2)) = (1 - t)^{-2}$, $t < 1$.

13 a. Let x be $n(\mu, \sigma^2)$ so that $\Pr(x < 89) = 0.90$ and $\Pr(x < 94) = 0.95$. Find μ and σ^2.

(*Or*)

b. Let x have a poisson distribution with parameter $\mu > 0$. If k is a non-negative integer and if $\mu^1_k = E(x^k)$, Prove that

$$\mu'_{(k+1)} = \mu\left(\mu'_k + \frac{d\mu'_k}{d\mu}\right).$$

14 a. Let x_1 and x_2 be two stochastically independent random variables that have gamma distribution and joint p.d.f.

$$f(x_1, x_2) = \frac{1}{\Gamma(\alpha)\,\Gamma(\beta)}\, x_1^{\alpha-1} x_2^{\beta-1} e^{-x1\,-x2}, \; 0 < x_1 < \infty, \; 0 < x_2 < \infty,$$

Zero else where, where $\alpha > 0$, $\beta > 0$. Find the marginal p.d.f. of $x_1/(x_1 + x_2)$

(*Or*)

b. Let x_1, x_2 be a random sample from a distribution having the p.d.f. $f(x) = e_a^{-x}$, $0 < x < \infty$, Zero elsewhere. Show that $Z = x_1 / x_2$ has a F - distribution.

15 a. Let Zn be $\chi^2(n)$. Obtain the limiting distribution of the random variable $y_n = \frac{Zn - n}{\sqrt{2n}}$

(*Or*)

b. Let S^2n denote the variance of a random sample of size n from distribution which is $\underset{x}{n}(\mu, \sigma^2)$. Prove that $\frac{nS^2}{(n-1)}$ converges stochastically to σ^2.

Part - C (3 × 10 = 30)

Answer any **Three** questions

16. a. State and prove chebyshev's inequality.

b. If x is a random variable such that $E(x) = 3$ and $E(x^2) = 13$, determine a lower bound for the probability $Pr(-2 < x < 8)$ using chebyshev's inequality.

17. a. State and prove Bayes' theorem.

b. Let x_1 and x_2 have the joint p.d.f. $f(x_1, x_2) = x_1 + x_2$, $0 < x_1 < 1$, $0 < x_2 < 1$, Zero else where. Find the conditional mean an variance of x_2 given $x_1 = x_1$, $0 < x_1 < 1$.

18. a. Derive the poisson distribution as the limiting form of binomial distribution.

b. If the random variable x is n (μ, σ^2), $\sigma^2 > 0$, Prove that the random variable $v = \frac{(x - \mu)^2}{\sigma^2}$ is $\chi^2(1)$.

19. Define the F-variate. Obtain the p.d.f. of a F-variate.

20. State and prove the central limit theorem.

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AF-1893 **MMA2E1**

M.Sc. DEGREE EXAMINATION, APRIL 2010

Second Semester

Mathematics

ELECTIVE - APPLIED ALGEBRA

(CBCS—2008 Onwards)

Duration : 3 Hours Maximum : 75 Marks

Part - A $(10 \times 2 = 20)$

Answer **all** Questions

1. Define a Turning machine and give an example.

2. Define (i) Covering Machines
   (ii) Equivalent machines.

3. Write ALGOL expressions for the following :
   (i) Sin 4 $(A + 3^2)$
   (ii) $(A^N - B^N)(C - D)$

4. Evaluate the following ALGOL expressions
   (i) $A - B + C \uparrow D \times E$
   (ii) $A \uparrow D - E \uparrow F$
   When A = 2, B = 3, C = -2, D = 4, E = 5 and F = 6.

5. Prove that the idempotent laws follow from absorption laws.

6. Is $(p \to q) \to r = (p \to r) \to q$, a tautology of a Boolean algebra ? Justify.

7. Prove that any subpath of an optimal path is optimal.

8. When do you say
   (i) a product term α subsumes a product term β
   (ii) a product term α is a completion of a term β?

9. What is a group code ? Give an example.

10. Define a Hamming Code and give an example.

Part - B (5 × 5 = 25)

Answer **all** Questions

11 a. Establish a morphism from M to $\overline{M}$ where

	γ	ε
M	0 1	0 1
a	b c	0 1
b	a c	1 0
c	c a	1 0

	γ	ε
$\overline{M}$	0 1	0 1
1	1 2	0 1
2	2 1	1 0

(Or)

b. Minimize the number of states in the following machine :

	Next State		Out put	
	a_0	a_1	a_0	a_1
1	2	2	1	0
2	3	3	1	0
3	4	4	1	0
4	4	4	0	1
5	5	6	1	1
6	6	5	1	1

12 a. Write the general form of for statement in ALGOL. Write a program in ALGOL to find the sum of cubes $1^3 + 2^3 + \ldots + n^3$.

(Or)

b. Let A be a nxn matrix. Write an ALGOL program that multiples each aij in the matrix A by 6 if i = j and leaves the element as it is if $i \neq j$.

13 a. In a Boolean algebra, prove that the Boolean sum (Symmetric difference) is an associative operation.

(Or)

b. Prove that each Boolean expression in $y = \{ y_1, y_2, \ldots, y_n\}$ can be put into the disjunctive normal form $V_s p(N)$ where

$$P(w) = \hat{n}\, zi,\ zi = \begin{cases} yi \text{ if } w_i = 1 \\ \hat{yi} \text{ if } w_i = 0 \text{ in one} \end{cases}$$

14 a. Prove that any Boolean function of n variables can be expressed in product of sums form.

(Or)

b. Find a minimal sun - of - products expression for $f(a, b, c) = \Sigma (0, 3, 5, 6)$ where $0 = a^1 b^1 c^1$, $3 = a^1 b c$, $5 = a b^1 c$.

15 a. For a code to detect all sets of k or fewer errors prove that it is necessary and sufficient that the minimum distance between any two code words be k + 1 or more.

(Or)

b. Prove the following :

(i) Any group translation $x \rightarrow x + c$ of the group $(Z_2 r , +)$ preserves distance between code words.

(ii) In a binary group code, every code word is at the same minimum distance from the set of the other code words.

Part - C (3 × 10 = 30)

Answer any **three** Questions

16. Consider the following machine : M = {A, S, γ, z, ε} and

	γ	ε
M	0 1	0 1
1	1 2	0 1
2	1 3	0 1
3	5 1	0 1
4	4 2	0 1
5	4 3	1 1

Where A = {0, 1}, z = {0, 1}, S = 1, 2, 3, 4, 5}.

a) Find a Machine $\overline{M}$ with the minimal number of states which is output destinguishable from M for all input tapes of length 2.

b) Is your machine $\overline{M}$ equivalent to M ? Justify.

17. a) Write an ALGOL program to compute

$$F(x) = \begin{cases} 3125 - x^x & \text{if } x < 5 \\ (x - 5) / (1 + x^2) & \text{if } x \geq 5 \text{ for x ranging} \end{cases}$$

from 1 to 10 in steps of 0.5

b) Write a program in ALGOL to find

(i) The mean and

(ii) The standard deviation of 50 values.

18. a) Prove that $(x \wedge y) \vee (y \wedge z) \vee (z \wedge x) = (x \vee y) \wedge (y \vee z) \wedge (z \vee x)$ in any distributive lattice.

b) Prove that $xy \vee xz \vee zy^1 = xy \vee z^1y^1$ and simply the Boolean expression $(x\,y \vee x)\,(z^1 \vee z\,x)$.

19. Using optimality principle, write an algorithm to find the shortest path from a node a to a node b in any directed graph $\overline{G}$.

20. Explain with an example the procedure for forming a Hamming code and finding the transmitted code.

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AF-1894 MMA 2E2

M.Sc DEGREE EXAMINATION, APRIL 2010

II Semester

MATHEMATICS

ELECTIVE: PROGRAMMING IN JAVA

(CBCS - 2008 Onwards)

Duration : 3 Hours Maximum : 60 marks

Part - A $(10 \times 1\frac{1}{2} = 15)$

Answer ALL Questions

1. What are the assignment operators that are available in JAVA? What are their uses?

2. Write the general form of if .... else statement in JAVA. Write about its execution.

3. What are objects? How are they created?

4. How is a method defined?

5. What is a thread? How do we start a thread?

6. What Java interface must be implemented by all threads?

7. What is a local applet? Give an example.

8. What is a remote applet? Give an example.

9. What is a URL? What are its components?

10. What are the functions of

    i. parse ( )

    ii. to string ( ) methods.

Part-B (5 x 3 = 15)

Answer ALL Questions

11. a. Given a five digit number, write a program using while loop to reverse the digits of the number.

(OR)

b. Write a program to convert a given integer in decimal form to its binary equivalent.

12. a. Explain with an example multilevel inheritance.

(OR)

b. What is a constructor? How do we invoke a constructor? What are its special properties?

13. a. What is the difference between multiprocessing and multi-threading? What are to be done to implement these in a program?

(OR)

b. What is a thread? Explain the complete life cycle of a thread.

14. a. Discuss the steps involved in loading and running a remote applet.

(OR)

b. How do applets differ from application programs? Explain.

15. a. Write a source code for Url Cache entry.

(OR)

b. Write a source code for Mime Header.

Part-C (3 x 10 = 30)

Answer any THREE Questions

16. Write a program to find the mean and standard deviation of 50 numbers.

17. Write a class COMPLEX to represent a complex number. The class should contain methods to receive data, to display data, to add two complex numbers, to multiply two complex numbers. Write a main ( ) program to test your class.

18. Illustrate the use of multithreads by a suitable application program.

19. Develop an applet that receives three numeric values, the coefficient a, b, c of the quadratic equation $ax^2 + bx + c = 0$, from the user and then displays the roots of the equation on the screen. Write a HTML page and test the applet.

20. Write an applet which makes an instance of the httpd, passing in itself as the Log Message Interface. It must create a panel that has a simple label at the topt, a Text Area in the middle for displaying the Log Messages and a panel at the bottom that has two buttons and another label in it. Any time a message is logged, the bottom-right label object must be updated to contain the latest statistics from the httpd.

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AF-1895

MMA2E3

M.Sc. DEGREE EXAMINATION, APRIL 2010

Second Semester

Mathematics

ELECTIVE - CODING THEORY

(CBCS—2008 Onwards)

Duration : 3 Hours

Maximum : 75 Marks

Part - A $(10 \times 2 = 20)$

Answer **All** Questions

1. Find the weight of 10011.

2. what is the code word representing 101 under (3, 4) parity check ?

3. What is the distance of an (n, k) linear code ?

4. Write down the parameters of the dual of (n, k) linear lock code.

5. Define a perfect code.

6. Define a cyclic code.

7. How many code words are there in any perfect code of length and distance $2t + 1$?

8. What do you mean by C_{24} ?

9. Write down the parameters of Hamming Codes.

10. Write down the parameters of BCH codes.

Part - B $(5 \times 5 = 25)$

Answer **All** Questions

11 a. Show that (m, m+1) parity check code $C : B^m \rightarrow B^{n+1}$ is a group code.

(Or)

b. Check whether S = {110, 011, 101, 111} is linearly independent or not.

12 a. List the cosets of the linear code C = {0000, 1001, 0101, 1100}.

(Or)

b. Consider a (7, 4) linear code whose generator matrix is

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & : & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & : & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & : & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & : & 0 & 1 & 1 \end{bmatrix}$$

i) Find all code vectors of this code.

ii) Find the parity check matrix for this code.

13 a. Form a parity check matrix for a (5, 11) systematic hamming code.

(Or)

b. For / $H = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ find $C_H = B^3 \to B^6$

14 a. Give the code word assignments for the encoding polynomial $1 + x^2 + x^3$.

(Or)

b. Find the generator polynomial for the smallest linear cyclic codes containing that word for the given word 0 1 0 0 1 0.

15 a. Discribe BCH code.

(Or)

b. Show that Hamming Codes are a special cas of BCH codes.

Part - C (3 × 10 = 30)

Answer any **Three** Questions

16. Construct the IMLD table for the following codes

i) {000, 001, 010, 011}

ii) {00000, 11111}

17. Explain the concept of coset leaders in coding theory.

18. a) For only (n, k, d) linear code, prove that $d-1 \leq n-k$

b) Consider the linear code whose generator matrix is $\begin{bmatrix} 1\,0\,1 & 0\,1\,1 \\ 0\,1\,1 & 1\,1\,0 \\ 0\,0\,0 & 1\,1\,1 \end{bmatrix}$ Find a generator matrix G in standard form for an equivalent code.

19. a) Prove that g(x) is the generator polynomial for a linear cyclic code of length n if and only if, g(x) devides $1 + x^n$.

b) Let $g(x) = 1 + x^2 + x^3$ be the generator polynomial of a linear cyclic code of Length ? Encode the following polynomials $1 + x^3$, x.

20. a) Prove that binary BCH code with code word length $2^m - 1$ and with minimum distance d can always be constructed with [(d-1/2]m or fewer check symbols.

b) Consider with BCH code generated over z_d by

$p(x) = 1 + x + x^2 + x^4 + x^5 + x^8 + x^{10}$. Show that p(x) generates (5, 15,) group code.

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AF 1896	MMA3C1

M.Sc. DEGREE EXAMINATION
APRIL 2010
Third Semester
Mathematics
Elective – COMPLEX ANALYSIS
(CBCS – 2008 onwards)

Time : Three Hours   Maximum : 75 Marks

Part - A   (10 × 2 = 20)

Answer **All** *questions.*

1. Verify C.R. equations for the function $f(z) = z^3$.

2. Prove that the functions $u\ (z)$ and $u(\bar{z})$ are simultaneously harmonic.

3. Evaluate $\int_c f(z)\, dz$ where $f(z) = y - x - i\ 3x^2$ and C is the line segment from $z = 0$ to $z = 1 + i$.

4. Evaluate $\int_c \frac{z\,dz}{z^2 - 1}$ where c is the positively oriented circle $|\ z\ | = 2$.

5. Prove that f (z) = $e^{1/z}$ has a removable singularity at $z = 0$.

6. Prove that $f\ (z) = \frac{z - \sin z}{z^3}$ has a removable singularity at $z = 0$.

7. Evaluate $\int_c \frac{dx}{2z+3}$ where C is $|\ z\ | = 2$.

8. Evaluate $\int_c \frac{e^{2z}\, dz}{(z+1)^3}$ where C is $|z| = \frac{3}{2}$.

9. What is the coefficient of z^7 in the Taylor development of $\tan z$?

10. Develop $\frac{1}{(1+z)^2}$ in powers of $(z - a)$, a being a real number.

Part - B (5 × 5 = 25)

Answer **All** *questions.*

11. (a) Find a linear transformation which carries $|z| = 1$ and $\left| z - \frac{1}{4} \right| = \frac{1}{4}$ into concentric circles. What is the ratio of the radii?

Or

(b) Find the bilinear transformation which maps the points 2, i, – 2 onto 1, i, –1 respectively.

12. (a) Obtain a necessary and sufficient condition for the line integral $\int_{\gamma} p\,dx + q\,dy$ defined in Ω to depend only on the end point of γ.

Or

(b) State and prove Cauchy's integral formula.

13. (a) Prove that an analytic function comes arbitrarily close to any complex value in every neighbourhood of an essential singularity.

Or

(b) State and prove Local mapping theorem.

14. (a) State and prove Residue theorem.

Or

(b) State and prove argument principle.

15. (a) State and prove Jensen's formula.

Or

(b) Prove that the infinite product $\overset{\infty}{\underset{1}{\pi}} (1 + a_n)$ with $1 + a_n \neq 0$ converges simultaneously with the series $\sum\limits_{1}^{\infty} \log (1 + a_n)$ whose terms represent the values of the principal branch of the logarithm.

Part -C (3 × 10 = 30)

Answer any **three** *questions.*

16. (a) Prove that a linear transformation carries circles into circles.

(b) State and prove Luca's theorem.

17. State and prove Cauchy's theorem for a rectangle.

18. State and prove Taylor's theorem.

19. Evaluate :

(a) $\int\limits_{0}^{\infty} \dfrac{\cos x \, dx}{x^2 + a^2}$ where a is real.

(b) $\int\limits_{0}^{\infty} \dfrac{x^2 \, dx}{x^4 + 5x^2 + 6}$.

20. State and prove Weierstrass theorem on the existence of an entire function with arbitrarily prescribed zeroes.

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AF-1897 | MMA3C2

M.Sc. DEGREE EXAMINATION
APRIL 2010
Third Semester
Mathematics
TOPOLOGY – I
(CBCS – 2008 onwards)

Time : Three Hours | Maximum : 75 Marks

Part -A $(10 \times 2 = 20)$

Answer **all** *questions.*

1. Define a topology on a set X. Give two examples of topologies on $X = \{a, b, c\}$.

2. Define product topology and give an example.

3. Define a continuous function and give an example.

4. Define a metric space and give an example.

5. Define a connected space. Is Q, the set of rationals connected? Justify.

6. Define a compact space. Is R, the set of reals, a compact space? Justify.

7. Define components and path components of a space.

8. Is the finite union of compact sets compact? Justify.

9. Give an example of a Hausdorff space which is not normal.

10. Define a first countable space and give an example.

Part - B $(5 \times 5 = 25)$

Answer **All** *questions.*

11. (a) Let X be a topological space. Suppose that $\mathcal{C}$ is a collection of open sets of X such that for each x in X and each open set U of X, there is an element C of $\mathcal{C}$ such that $x \in C \subset U$. Prove that $\mathcal{C}$ is a basis for the topology of X.

(*Or*)

(b) Prove that the product of two Hausdorff spaces is a Hausdorff space and the subspace of a Hausdorff space is a Hausdorff space.

12. (a) Let X and Y be topological spaces and $f : X \to Y$. Prove that the following are equivalent :–

(i) f is continuous.

(ii) for every subset A of X, $f(\bar{A}) \subseteq \overline{f(A)}$.

(iii) for every closed set B in Y, $f^{-1}(B)$ is closed in X.

(*Or*)

(b) Let X be a metric space with metric d. Define $\bar{d} : X \times X \to R$ by $\bar{d}(x, y) = \min\{1, d(x, y)\}$. Prove that $\bar{d}$ is a metric that induces the topology of X.

13. (a) Prove that the image of a connected space under a continuous map is connected.

(*Or*)

(b) Construct an example of a connected space that is not path connected.

14. (a) Prove that a compact subset of a Hausdorff space is closed.

(*Or*)

(b) Let X be a non-empty compact Hausdorff space. If every point of X is a limit point of X, prove that X is uncountable.

15. (a) Prove that every compact Hausdorff is normal.

(*Or*)

(b) Prove that the product of regular spaces is regular.

Part - C $(3 \times 10 = 30)$

Answer any **Three** *questions.*

16. Prove that the topologies on R^n induced by the euclidean metric d and the square metric ρ are the same as the product topology on R^n.

17. (a) Let X, Y, Z be topological spaces. If $f : X \to Y$ and $g : Y \to X$ are continuous prove that $gof : X \to Z$ is continuous.

(b) State and prove uniform limit theorem.

18. If L is a linear continuum in the order topology prove that L is connected.

19. Let X be a metrizable space. Prove that the following are equivalent :

    (a) X is compact.

    (b) X is limit point compact.

    (c) X is sequentially compact.

20. State and prove Urysohn's lemma.

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M.Sc. DEGREE EXAMINATION, APRIL 2010

Third Semester

Mathematics

GRAPH THEORY

(CBCS—2008 Onwards)

Duration : 3 Hours Maximum : 75 Marks

Part - A $(10 \times 2 = 20)$

Answer **All** Questions

1. Define a simple graph and give an example.

2. Prove that the number of vertices of odd degree is even in any graph.

3. If G is connected graph, with usual notations, prove that $\varepsilon \geq \gamma - 1$.

4. Define a cutvertex of a graph and give an example.

5. Define a perfect Matching and give an example.

6. Define edge chromatic number of a loopless graph G. Give an example of a graph which 3-edge chromatic.

7. Define Ramsey numbers $r(k, l)$ and write the value of $r(3, 3)$.

8. In a critical graph, prove that no vertex cut is a lique.

9. Define a Planar graph and give an example of a non-planar graph.

10. If G is a simple, planar graph with $\varepsilon \leq \gamma - 6$.

Part - B $(5 \times 5 = 25)$

Answer **All** Questions

11. a. Let G be a graph with $\gamma - 1$ edges. Prove that the following are equivalent :

    i) G is connected.

    ii) G is a cyclic.

    iii) G is a tree.

    (*Or*)

    b. Obtain a necessary and sufficient condition for an edge *e* of G to be a cut edge of G.

12. a. Find a necessary and sufficient condition for a graph G with $\gamma \geq 3$ to be 2-connected. Deduce that if G is 2-connected, then any two vertices of G lie on a common cycle.

    (*Or*)

    b. Define a Hamiltonian cycle and a Hamiltonian path. If G is Hamiltonian, prove that for every non-empty proper subset S of V, $w\,(G/S \leq |\,s\,|$.

13. a. Let G be a bipartite graph with bipartition (X, Y). Prove that G contains a matching that saturates every vertex in X if, and only if, $|N(S)| \geq |S|$ for all $S \subseteq X$.

(*Or*)

b. If G is bipartite, prove that $X' = \Delta$.

14. a. In a graph G, if $\delta > 0$, prove that $\alpha' + \beta' = \gamma$.

(*Or*)

b. Show that X $(G_1, VG_2) = X(G_1) + X(G_2)$ and $G_1 V G_2$ is critical if, and only if both G_1 and G_2 are critical.

15. a. Let G be a non-planar connected graph that contains no sub division of K_5 or $K_{3,3}$ and has a few edges as possible. Prove that G is simple and 3-connected.

(*Or*)

b. State and prove Euler's formula for connected plane graphs.

Part - C $(3 \times 10 = 30)$

Answer any **Three** Questions

16. a. With usual notations, prove that $\tau(K_n) = n^{n-2}$.

b. Prove that a graph G is bipartite if, and only if, G contains no odd cycle.

17. a. Show that the closure of a graph C (G) is well-defined.

b. Prove that a simple graph G is Hamiltonian if, and only if, its closure is Hamiltonian.

18. Prove that a graph G has a perfect matching if, and only if, $0\,(G \setminus S) \leq |S|$ for all $S \subseteq V$.

19. a. State and prove Dirac's theorem.

b. State and prove Brook's theorem.

20. State and prove Kuratowski's theorem.

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5. Write three axioms in exponential distribution.

6. What is the truncated Poison distribution in the pure death model ?

7. What is little's formula ?

8. Write a short note on multiple - server models.

9. Explain separable programming with an example.

10. What is direct search method ?

Part - B $(5 \times 5 = 25)$

Answer **All** the questions.

11. (a) The Midwest TV cable company is in the process of providing cable service to five new housing development areas. The following figure depicts the potential TV linkages among the five areas. The cable miles are shown on each branch. Determine the most economical cable network.

robability that no students will arrive at the
uring the next 15 minutes ?

robability that at least one student will visit
n during the next 20 minutes.

t Ozark college is limited to only five spaces.
of this space arrive according to a Poisson
e rate of six cars per hour. Parking time is

17. Find the optimal solution for the
situation given subsequently. The d
units, and the starting inventory
production cost is $ 10 for the first
additional unit.

Period i	Demand D_i	Set up K_i

12. (a) Lube car specializes in fast automobile oil change. The garage buys car oil in bulk at \$ 3 per gallon. A price discount of \$ 2.50 per gallon is available if Lube car purchases more than 1,000 gallons. The garage services approximately 150 cars per day and each oil change requires. 1.25 gallons. Lube car stores bulk oil at the cost of \$.20 per gallon per day. Also, the cost of placing an order for bulk oil is \$ 20. There is a 2- day lead time for delivery. Determine the optional inventory policy.

(b) What is the pr

game room du

(c) What is the pr

the game roon

19. Visitors parking a

Cars making use

distribution at th

three - period inventory

lemand occurs in discrete

is $x_1 = 1$ unit. The unit

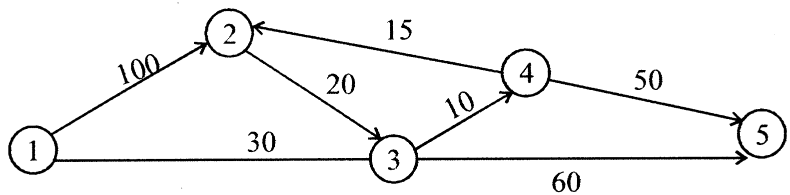
3 units and \$ 20 for each

cost	Holding cost h_i

Month		Capacity Regular (Units)	Overtime (Units)	Demand (Units)
1	...	90	50	100
2	...	100	60	190

(*Or*)

(b) The network in the following figure gives the permissible routes and their lengths in miles between city 1 and four other cities. Determine the shortest routes from city 1 to each of the remaining four cities.



($)
1
3
2

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≤ 3

game room in the student

minutes.

r ?

3	...	120	80	210
4	...	110	70	160

The Unit production cost in any period is \$6 during regular time and \$ 9 during overtime. The holding cost per unit per month is \$.10.

13. (a) Babies are born in a sparsely populated state at the rate of one birth every 12 minutes. The time between births follows an exponential distribution. Find the following :

(i) The average number of births per year.

	(Units)	($)
1	3	3
2	2	7
3	4	6

The production cost function for p

$C_i\,(Zi) = K_i + C_i\,(Z_i)$ for $Z_i > 0$, v

$$C_i\,(Z_i)\begin{cases}10\,Z_i & 0 \le Z_i \\ 30 + 20\,(Z_i - 3), & Z_i \ge 4.\end{cases}$$

18. The time between arrivals at the g union is exponential with mean 10 r

(a) What is the arrival rate per hou

AF-1900 **MMA3E1**

M.Sc. DEGREE EXAMINATION, APRIL 2010

Third Semester

Mathematics

Elective : COMBINATORIAL MATHEMATICS

(CBCS—2008 Onwards)

Duration : 3 Hours Maximum : 75 Marks

Part - A (10 × 2 = 20)

Answer **All** the questions.

1. Prove that

$$\binom{n}{0}^2 + \binom{n}{1}^2 + \binom{n}{2}^2 + \ldots + \binom{n}{r}^2 + \ldots + \binom{n}{n}^2 + \binom{2n}{n}.$$

2. What is the coefficient of the term x^{23} in $\left(1 + x^5 + x^9\right)^{100}$?

3. Solve the difference equation $a_n + 2a_{n-1} + a_{n-2} + 2^n$.

4. Define the Fibonacci numbers and form the recurrence relation.

5. What do you mean by a derangement of the integers 1, 2, ..., n ? Write the number of derangements of n integers.

6. Let n books be distributed to n children. The books are returned and distributed to the children again later on. In how many ways can the books be distributed so that no child will get the same book twice ?

7. Find the number of distinct strings of length 3 that are made up of blue leads and yellow leads.

8. Find the binary relation induced by G where,

$$G = \left\{ \begin{pmatrix} a, b, c, d \\ a, b, c, d \end{pmatrix}, \begin{pmatrix} a, b, c, d \\ b, a, c, d \end{pmatrix}, \begin{pmatrix} a, b, c, d \\ a, b, d, c \end{pmatrix}, \begin{pmatrix} a, b, c, d \\ b, a, d, c \end{pmatrix} \right\}.$$

9. Define a block design and give an example.

10. Define a Hadamard matrix and give an example.

Part - B (5 × 5 = 25)

Answer **All** the questions.

11. (a) Find the number of r-digit quaternary sequences that contain an even number of 0's.

(*Or*)

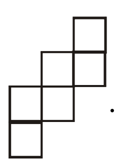
(b) Evaluate the sum $1^2 + 2^2 + ... + r^2$.

12. (a) Solve the difference equation $a_n + 2a_{n-1} = n + 3$.

(*Or*)

(b) Find the number of ways to parenthesize the expression $w_1 + w_2 + ... w_{n-1} + w_n$, so that only two terms will be added to one at one time.

13. (a) Define the rook polynomial and find the rook polynomial for



(*Or*)

(b) Find the number of r - digit quaternary sequences in which each of the three digits 1, 2 and 3 appears at least once.

14. (a) Find the number of distinct bracelets of five beads made up of yellow, blue and white beads.

(*Or*)

(b) Find the number of ways of painting the four faces a, b, c and d of a pyramid with two colours x and y.

15. (a) In a block design prove that each element lies in exactly r blocks where $r(k-1) = \lambda(v-1)$ and $bk = vr$.

(*Or*)

(b) Prove that there are no integers a, b, c, such that $a^2 + b^2 = 6c^2$, apart from $a = b = c = 0$.

Part - C (3 × 10 = 30)

Answer any **Three** questions.

16. (a) Find the number of n-digit words generated from the alphabet $\{0, 1, 2\}$ in each of which none of the digits appears exactly three times.

(b) In how many ways can 200 identical chairs be divided among four conference rooms such that each room will have 20 or 40 or 60 or 80 or 100 chairs ?

17. (a) Find the number of n-digit binary sequences that have the pattern 010 occurring at the n^{th} - digit.

(b) Find the number of r - combinations of n - distinct objects with unlimited repetitions.

18. If d_n is number of derangement of n objects prove that $d_n - nd_{n-1} = (-1)^n$ and hence find an expression for d_n.

19. State and prove Polya's fundamental theorem.

20. For a (b, v, r, k, λ) – configuration, prove that $b \geq v$.

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AF-1901 **MMA3E2**

M.Sc. DEGREE EXAMINATION, APRIL 2010

Third Semester

Mathematics

Elective—STOCHASTIC PROCESS

(CBCS—2008 Batch)

Duration : 3 Hours Maximum : 75 Marks

Part - A $(10 \times 2 = 20)$

Answer **All** questions.

1. State the condition, under which a stochastic process $\{X(t), t \in T\}$ will become a Markov Process.

2. Define a Gaussian Process.

3. Define the transition probability matrix of the Markov chain.

4. What is an irreducible chain.

5. Define an intree T_j to a specific point j in a directed graph G.

6. State Solberg theorem.

7. Let $N_1(t)$ and $N_2(t)$ be two independent Poisson processes. Write down the probability distribution of $N(t) = N_1(t) - N_2(t)$.

8. If customers arrive at a counter in accordance with a Poisson process with a mean rate of 2 per minute, find the probability that the interval between 2 consecutive arrivals is more than one minute.

9. Define a renewal process and give an example for it.

10. What do you mean by a stopping time for the sequence of random variable $\{X_i\}$?

Part - B (5 × 5 = 25)

Answer **All** questions.

11. (a) Prove that the process $X(t) = A_1 + A_2 t$ where A_1 and A_2 are independent random variables with $E(A_i) = a_i$ var $(A_i) = \sigma_i^2$, $i = 1,2$ is evolutionary.

(*Or*)

(b) Prove that the process $X(t) = A \cos wt + B \sin wt$ where A and B are uncorrelated random variables each with mean 0 and variance 1 is covariance stationary (w is a positive constant).

12. (a) Let the probability of a dry day (state 0) following a rainy day (state 1) is $\frac{1}{3}$ and that the probability of a rainy day following a dry day is $\frac{1}{2}$. Given that May 1st is a dry day. Find the probability that May 5th is a dry day.

(*Or*)

(b) If state k is either transient or persistent null, then prove that for j, $p_{jk}^{(n)} \to 0$ as $n \to \infty$.

13. (a) If the state j is persistent, then for every state k that can be reached from the state j, Prove that $F_{kj} = 1$.

(*Or*)

(b) For the Markov chain having t.p.m.

$$\begin{pmatrix} 0 & 2/3 & 1/3 \\ 1/2 & 0 & 1/2 \\ 1/2 & 1/2 & 0 \end{pmatrix}$$, show that $v_1 = \frac{1}{3}$,$v_2 = \frac{10}{27}$ and

$v_3 = \frac{8}{27}$.

14. (a) If N(t) is a Poisson process and $s < t$, show that

$$\Pr\{N(s) = k/N(t) = n\} = \binom{n}{k}\left(\frac{s}{t}\right)^k \left(1-\left(\frac{s}{t}\right)\right)^{n-k}$$

(*Or*)

(b) Prove that the interval between two successive occurrences of a Poisson process $\{N(t)\}$ having parameter λ has a negative exponential distribution with mean $\frac{1}{\lambda}$.

15. (a) Prove that the renewal function M satisfies the equation.

$$M(t) = F(t) + \int_0^t M(t-x)\, d\, F(x).$$

(*Or*)

(b) Prove that with probability 1,

$$\frac{N(t)}{t} \to \frac{1}{-} \text{ as } t \to \infty, \quad = (X_n) \leq \infty \quad .$$

Part - C (3 × 10 = 30)

Answer any **three** the questions.

16. (a) Let X_n for n even take values +1 and –1 with each with probability $\frac{1}{2}$ and for n odd take values $\sqrt{a}$, $-\frac{1}{\sqrt{a}}$ with probability $1/(a+1)$, $a/(a+1)$ respectively. (a is a real number > -1 and $\neq 0,1$). Further let X_n be independent. Show that $\{X_n\}$ is covariance stationary.

(b) Explain Polya's urn model.

17. Suppose that a fair die is tossed. Let the states of X_n be k (=1, 2,...., 6) where k is the maximum number shown in the first n tosses. Find P and P². Also calculate $\Pr(X_2 = 6)$.

18. For an irreducible ergodic chain, show that the limits $v_k = \lim_{n \to \infty} p_{jk}^{(n)}$ exist and are independent of the initial state j. Also the limits v_k are such that $v_k \geq 0, \sum v_k = 1$. Furthermore the limiting probability distribution $\{ v_k \}$ is identical with the stationary distribution for the given chain so that

$$v_k = \sum_j v_j p_{jk} \ , \sum v_k = 1.$$

19. For the counting process N(t) under independence, Homogeneity and regularity. Obtain the Poisson law that

$$P_n(t) = P_r \{N(t) = n\} = \frac{e^{-\lambda t} (\lambda t)^n}{\lfloor n}, \ n = 0, 1, 2, \ldots .$$

20. State and prove the Elementary Renewal theorem.

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AF 1902 | MMA3E3

M.Sc. DEGREE EXAMINATION APRIL 2010

Third Semester

Mathematics – Elective – Course – III (C)

FUZZY MATHEMATICS

(CBCS – 2008 onwards)

Time : Three Hours | Maximum : 75 Marks

Part -A $(10 \times 2 = 20)$

Answer **all** *questions.*

1. Define a normalized fuzzy set and give an example.
2. Define the union of two fuzzy sets and give an example.
3. Define complement of a fuzzy set and give an example.
4. For all $a, b \in [0, 1]$, prove that $i(a, b) \leq \min(a,b)$.
5. Define a symmetric fuzzy relation and give an example.
6. Define a proximity relation and give an example.
7. Define a belief measure.
8. State the axioms of fuzzy measure.
9. Define a vector-maximum problem.
10. Define an optimal compromise solution of a vector maximum problem.

Part -B $(5 \times 5 = 25)$

Answer **all** *questions.*

11. (a) Compute the scalar cardinality and the fuzzy cardinality for the fuzzy sets.

    (i) $A = \frac{1}{x} + \frac{1}{y} + \frac{1}{z}$;

    (ii) $\mu_c(x) = \frac{x}{x+1}$, $x \in \{0, 1, 2, \ldots\ldots, 10\}$.

Or

(b) Determine $A \cup B, A \cap B, \bar{A}, \bar{B}$ for the fuzzy sets A and B defined on the interval X = [0, 10] of real numbers by the membership functions $\mu_A(x) = \frac{x}{x+2}$,

$$\mu_B(x) = \frac{1}{1 + 10(x-2)^2}.$$

12. (a) Prove that $u(a, b) = \max(a, b)$ is the only continuous and idempotent fuzzy set union.

Or

(b) Show that the Fuzzy set operations of union, intersection and continuous complement that satisfy the law of excluded middle and the law of contradiction are not idempotent or distributive.

13. (a) Determine the transitive max – min closure R_T (x, x) for a fuzzy relation R (x, x) defined by the membership matrix $\begin{bmatrix} \cdot7 & \cdot5 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & \cdot4 & 0 & 0 \\ 0 & 0 & \cdot8 & 0 \end{bmatrix}$.

Or

(b) Solve the following fuzzy relation equation

$$P \bullet \begin{bmatrix} \cdot9 & \cdot6 & 1 \\ \cdot8 & \cdot8 & \cdot5 \\ \cdot6 & \cdot4 & \cdot6 \end{bmatrix} = [\cdot6 \ \cdot6 \ \cdot5].$$

14. (a) Obtain a necessary and sufficient condition for a belief measure Bel on a finite power set to be a probability measure.

Or

(b) For any given basic assignment m prove that the function Pl determine by $\mathrm{Pl}(A) = \sum_{A \cap B \neq \phi} m(B)$ for all $A \in \wp(x)$, is a plausibility measure.

15. (a) Explain with an example multi objective decision making (MOD M).

Or

(b) Explain with an example, Fuzzy dynamic programming with crisp state transformation function.

Part -C (3 × 10 = 30)

Answer any **three** *questions.*

16. (a) Show that all α - cuts of any fuzzy set A defined on R^n $(n \geq 1)$ are convex if, and only if, $\mu_A[\lambda r + (1-\lambda)s] \geq \min[\mu_A(r), \mu_A(s)]$ for all, r, s $\in R^n$ and all $\lambda \in [0, 1]$.

(b) Let the fuzzy sets A, B defined on the set X {0, 1, 2, ..., 10} by the membership grade functions

$\mu_A(x) = \frac{x}{x+2}$, $\mu_B(x) = \frac{1}{1+10(x-2)^2}$. Let $f(x) = x^2$ for all $x \in x$. Use the extension principle and derive f(A) and f(B).

17. (a) Prove that $\lim_{w \to \infty} \min[1, (a^w + b^w)^{1/w}] = \max$ (a, b).

(b) Show that $u_w(a, c_w(a) = 1$ for all $a \in [0, 1]$ and all $w > 0$ where u_w and c_w denote the Yager union and complement respectively.

18. (a) Given a fuzzy similarity relation R (x, x) and two partitions π (R_α) π (R_β) where R_α and R_β are α-cuts and $\alpha \geq \beta$, prove that each element of π (R_α) is contained in some element of π (R_β).

(b) Prove the following :

(i) When R (X, X) is a strictly antisymmetric crisp relation, then $R \cap R^{-1} = \phi$.

(ii) When R (x, x) is max-min transitive, then $R \circ R \subseteq R$.

19. Let $X = \{a, b, c, d, e\}$ and $y = N_8$. Using a joint possibility distribution on X xY given in terms of the matrix

$$\begin{array}{c} \\ a \\ b \\ c \\ d \\ e \end{array}
\begin{array}{c}
\begin{array}{cccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \end{array} \\
\begin{bmatrix}
1 & 0 & 0 & .3 & .5 & -2 & -4 & .1 \\
0 & .7 & 0 & .6 & 1 & 0 & .4 & .3 \\
0 & .5 & 0 & 0 & 1 & 0 & 1 & -5 \\
1 & 1 & 1 & .5 & 0 & 0 & 1 & .4 \\
-8 & 0 & .9 & 0 & 1 & -7 & 1 & .2
\end{bmatrix}
\end{array}$$

Determine (i) Marginal possibilities ; (ii) Joint and marginal basic assignments (iii) Both conditional probabilities of the two forms (iv) Hypothetical joint probability distributions based on the assumptions of non-intersection.

20. Consider the following problem :–

$$\text{Minimise } Z = 4x_1 + 5x_2 + 2x_3 \text{ such that}$$
$$3x_1 + 2x_2 + 2x_3 \leq 60,$$
$$3x_1 + x_2 + x_3 \leq 30,$$
$$2x_2 + x_3 \geq 10,$$
$$x_1, x_2, x_3 \geq 0.$$

Determine the optimal solution.

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AF-1903 **MMA4C1**

M.Sc. DEGREE EXAMINATION, APRIL 2010

Fourth Semester

Mathematics

FUNCTIONAL ANALYSIS

(CBCS—2008 Onwards)

Duration : 3 Hours Maximum : 75 Marks

Part - A $(10 \times 2 = 20)$

Answer **All** the Questions

1. Define a normed space. Give an example.

2. Give an example of a linear map which is not continuous.

3. Let X be a normed space over k, $f \in X^1$ and $f \neq 0$. Let $a \in X$ with $f(a) = 1$ and $r > 0$. Prove that $U(a, r) \cap z(f) = \phi$ if, and only if, $\|f\| \leq \frac{1}{r}$.

4. When do you say a normed space is a Banach Space ? Give an example.

5. Can a set of continuous functions from a metric space to a metric space be bounded at each point without being uniformly bounded ? Justify.

6. Let x, y, z be metric spaces. If $F : x \to y$ is continuous and $G : y \to z$ is closed prove that $GoF : x \to z$ is closed.

7. Define dual of a normed space. Give an example.

8. Define weak convergence of a sequence and give an example.

9. Does the Riesz representation theorem hold for incomplete inner product space ? Justify.

10. Let x be an inner product space. If $\|n_x - x\| \to 0$ and $\|y_n - y\| \to 0$ prove that $(x_n, y_n) \to (x, y)$.

Part - B (5 × 5 = 25)

Answer **all** the Questions

11 a. Let X be a normed space and Y be a subspace of X. If Y is finite dimensional prove that Y is complete.

(Or)

b. Let X and Y be normed spaces and $F : X \to Y$ be a linear map such that the range R(F) of F is finite dimensional. Prove that F is continuous if, and only if, the zero space Z(F) of F is closed in X.

12 a. State and prove Hahn-Banach extension theorem.

(Or)

b. Prove that a normed space X is a Banach space if, and only if, every absolutely summable series of elements in X is summable in X.

13 a. State and prove Resonance theorem.

(Or)

b. Let X be a normed space and $P : X \to X$ be a projection. Prove that P is a closed map if, and only if, the subspaces R(P) and Z(P) are closed in X.

14 a. State and prove the closed range theorem of Banach.

(Or)

b. Let X and Y be Banach spaces and $F \in BL(X, Y)$. Prove that $R(F) = y$ if, and only if, F^1 is bounded below.

15 a. State and prove Bessel's inequality.

(Or)

b. State and prove Projection theorem.

Part - C $(3 \times 10 = 30)$

Answer any **three** Questions

16. Let X denote a subspace of B(T) with the sup norm, $1 \in X$ and f be a linear functional on X. If f is continuous and $\|f\| = f(1)$, prove that f is positive. Conversely, if Re $x \in X$ whenever $x \in X$ and if f is positive, prove that f is continuous and $\|f\| = f(1)$.

17. a) Let X be a normed space and Y be a closed subspace of X. Prove that X is a Banach space if, and only if, Y and X/Y are Banach spaces in the induced norm and the quotient norm, respectively.

b) Let X and Y be normed spaces and $X \neq \{0\}$. Prove that BL (X, Y) is a Banach space in the operator norm if, and only if, Y is a Banach space.

18. State and prove closed graph Theorem.

19. State and prove Riesz representation theorem for C[a, b].

20. a) Let H be a non-zero Hilbert space over K. Prove that the following conditions are equivalent :

i) H has a countable orthonormal basis.

ii) H is Linearly isometric to k^n for some n or the l^2.

iii) H is separable.

b) Prove that a subset of a Hilbert space is weak bounded if, and only if, it is bounded.

AF-1904 **MMA4C2**

M.Sc. DEGREE EXAMINATION, APRIL 2010

Fourth Semester

Mathematics

NUMBER THEORY

(CBCS—2008 Onwards)

Duration : 3 Hours Maximum : 75 Marks

Part - A $(10 \times 2 = 20)$

Answer **all** the Questions

1. If $(a, b) = 1$ and $(a, c) = 1$ prove that $(a, bc) = 1$.

2. If $2^n - 1$ is a prime prove that n is a prime.

3. Define Möbius function μ and and find the value of $\sum_{d/100} \mu(d)$.

4. Define the Mangoldt function Λ and write the value of $\sum_{d/25} \Lambda(d)$.

5. With usual notations, prove that $[2x] - 2[x]$ is either 0 or 1.

6. If $x \geq 2$, prove that $\sum_{n < x} \frac{1}{\phi(n)} = O(\log x)$.

7. If $a \equiv b \pmod{m}$, prove that $(a, m) = (b, m)$.

8. Solve the congruence $25x \equiv 15 \pmod{120}$.

9. Write down the value of (3/11).

10. Write down the value of (2/13).

Part - B (5 × 5 = 25)

Answer **all** the Questions

11 a. State and prove the division algorithm.

(*Or*)

b. State and prove the fundamental theorem of arithmetic.

12 a. For $n \geq 1$, Prove that $\phi(n) = n \prod_{p/n} \left(1 - \frac{1}{p}\right)$

(*Or*)

b. Let f be multiplicative. Prove that f is completely multiplicative if, and only if, $f^{-1}(n) = \mu(n) f(n)$ for all $n \geq 1$.

13 a. Prove that the set of lattice points visible from the origin has density $\frac{6}{\pi^2}$

(*Or*)

b. For $x > 1$, prove that $\sum_{n \le x} \phi(n) = \frac{3}{\pi^2} x^2 + O(x \log x)$

14 a. State and prove Lagrange's theorem for polynomial congruences.

(*Or*)

b. Prove that $5n^3 + 7n^5 \equiv 0 \pmod{12}$ for all integers n.

15 a. State and prove Quadratic reciprocity law for Legendre symbol.

(*Or*)

b. Determine whether 888 is a quadratic residue or non residue of the prime 1999.

Part - C $(3 \times 10 = 30)$

Answer any **three** Questions

16. a) Use the Euclidean algorithm to compute $d = (826, 1890)$ and then express d as a linear combination of 826 and 1890.

    b) The sum of two positive integers in 5264 and their least common multiple is 200340. Determine the two integers.

17. Prove that the set of multiplicative functions is a subgroup of the group of all arithmetical functions.

18. For all $x \geq 1$, prove that $\sum_{n \leq x} d(n) = x \log x + (2c - 1)x + 0(\sqrt{x})$

where c is Euler's constant.

19. a) Find all x which simultaneously satisfy the system of congruence $x \equiv 1 \pmod 3$, $x \equiv 2 \pmod 4$, $x \equiv 3 \pmod 5$.

b) Let p be a prime, $p \geq 5$ and $1 + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{p} = \frac{r}{ps}$.

Prove that $p^3 / (r - s)$.

20. a) State and prove Euler's criterion for calculating (n/p).

b) Determine those odd primes for which $(-3/p) = +1$ and those for which $(-3/p) = -1$.

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AF-1905 MMA4C3

M.Sc. DEGREE EXAMINATION, APRIL 2010

Fourth Semester

Mathematics

ADVANCED STATISTICS

(CBCS—2008 Onwards)

Duration : 3 Hours Maximum : 75 Marks

Part - A $(10 \times 2 = 20)$

Answer **all** the Questions

1. Let the random variable X have the p.d.f. $f(x) = e^{-x}$, $0 < x < \infty$, zero elsewhere. Compute the probability that the random interval $(X, 3X)$ includes the point $x = 3$.

2. Let a random sample of size 17 from the normal distribution $n(\mu, \sigma^2)$ yield $\overline{x} = 4.7$ and $s^2 = 5.76$. Determine a 90% confidence interval for μ.

3. Prove that the sum of the items of a random sample of size n from a Poisson distribution having parameter θ, $0 < \theta < \infty$ is a sufficient statistic for θ.

4. Let $X_1, X_2, \ldots, X_n$ denote a random sample from a distribution with p.d.f.
$$f(x, \theta) = \begin{cases} \theta x^{\theta-1}, & 0 < x < 1. \\ 0 \text{ elsewhere}, & \theta > 0. \end{cases}$$
Find a sufficient statistic for θ.

5. Let $\overline{X}$ be the mean of a random sample of size n from a distribution which is $n(\theta, \sigma^2)$, $-\infty < \theta < \infty$, $\sigma^2 > 0$. Prove that for every $\sigma^2 > 0$, $\overline{X}$ is an efficient statistic for θ.

6. Let $X_1, X_2, \ldots X_n$ represent a random sample from a distribution that has p.d.f. $f(x, \theta) = \theta x^{\theta-1}, \theta < x < 1, 0 < \theta < \infty,$

$= 0$ elsewhere

Find the maximum likelihood statistic $\overline{\theta}$ for θ.

7. Define i) A statistical hypothesis and

ii) Critical region of a test.

8. Define a uniformly most powerful test.

9. Compute the mean and variance of a random sample which is $\chi^2 (r, \theta)$.

10. Show that the square of a non-central T random variable is a non-central F random variable.

Part - B (5 × 5 = 25)

Answer **all** the Questions

11 a. Let two independent random samples each of size 10, from two independent normal distributions $n(\mu_1, \sigma^2)$ and $n(\mu_2, \sigma^2)$ yield $\overline{x} = 4.8$, $s_1^2 = 8.64$, $\overline{y} = 5.6$, $s_2^2 = 7.88$. Find a 95% confidence interval for $\mu_1 - \mu_2$.

(Or)

b. If 8.6, 7.9, 8.3, 6.4, 8.4, 9.8, 7.2, 7.8, 7.5 are the observed values of a random sample of size 9 from a distribution that is $n(8, \sigma^2)$. Construct a 90% confidence interval for σ^2.

12 a. State and prove the Rao-Blackwell theorem.

(Or)

b. Let $X_1, X_2, \ldots, X_n$ denote a random sample of size $n > 2$ from a distribution with p.d.f.

$$f(x, \theta) = \theta e^{-\theta x},\ 0 < x < \infty$$
$$= 0 \quad \text{else where}$$

and $\theta > 0$. Prove that $Y = \sum_{i=1}^{n} X_i$ is a sufficient statistic for θ. Also prove that $(n-1)/Y$ is the best statistic for θ.

13 a. Let $X_1, X_2, \ldots, X_n$ be a random sample from the distribution having p.d.f. $f(x, \theta_1, \theta_2) = \frac{1}{\theta_2} e^{\frac{-(x-\theta_1)}{\theta_2}}$ $\theta_1 \leq x < \infty$, $-\infty < \theta_1 < +\infty$, $0 < \theta_2 < \infty$, zero else where. Find the maximum likelihood statistics for θ_1 and θ_2.

(Or)

b. Let $X_1, X_2, \ldots, X_n$ denote a random sample from a distribution which is $b(1, \theta)$, $0 < \theta < 1$. Find the decision function w which is a Baye's solution.

14 a. Let $X_1, X_2, \ldots, X_{15}$ denote a random sample from a distribution that is $n(0, \theta)$ where the variance θ is an unknown positive number. Test the simple hypothesis $H_0 : \theta = 3$, against the alternate composite hypothesis $H_1 : \theta > 3$, using uniformly most powerful test and obtain a best critical region of size 0.05.

(Or)

b. Let X have a Poisson distribution with mean θ. Find the sequential probability ratio test for testing $H_0 : \theta = 0.02$ against $H_1 : \theta = 0.07$.

Show that this test can be based upon the statistic $\sum_{i=1}^{n} X_i$.

If $\alpha_\alpha = 0.20$ and $\beta_\alpha = 0.10$, find $c_0(n)$ and $c_1(n)$.

15 a. Define a non central χ^2 variate and obtain its p.d.f.

(Or)

b. Let X_1 and X_2 be two stochastically indpendent random variables. Let X_1 and $Y = X_1 + X_2$ be $\chi^2 (r_1, \theta_1)$ and $\chi^2 (r, \theta)$ respectively, where $r_1 < r$ and $\theta_1 \leq \theta$. Prove that X_2 is $\chi^2 (r - r_1, \theta - \theta_1)$.

Part - C

(3 × 10 = 30)

Answer any **three** Questions

16. Two different teaching procedures were used on two different groups of students. Each group contained 100 students of about the same ability. At the end of the term, an evaluating team assigned a letter grade to each student. The results were tabulated as follows :

Grade	A	B	C	D	E	Total
Group I	15	25	32	17	11	100
Group II	9	18	29	28	16	100

If we consider these data to be observations from two independent multinomial distributions with k = 5, test as the 5 percent significant level, the hypothesis that the two distributions are the same.

17. State and prove the Fisher-Neyman theorem for the existence of a sufficient statistic for a parameter.

18. Let X_1, X_2, . . . , X_n denote a random sample from a Poisson distribution with parameter θ, $0 < \theta < \infty$. Let $Y = X_1 + X_2 + \ldots + X_n$ and let $Z(\theta, w(y)) = (\theta - w(y))^2$. If we restrict our considers the decision functions of the form $w(y) = b + y/n$ where b does not depend upon y, show that $R(\theta, w) = b^2 + \theta/n$. What decision function of this form yields a uniformly smaller risk than every other decision function of this form ? With this solution, say w and $0 < \theta < \infty$, determine $\max_{\theta} R(\theta, w)$ iif it exists.

19. State and prove Neyman - Pearson theorem.

20. Explain in detail one-way classification analysis of variance problem.

AF-1906 **MMA4C4**

M.Sc. DEGREE EXAMINATION, APRIL 2010

Fourth Semester

Mathematics

DISCRETE MATHEMATICS

(CBCS—2008 Onwards)

Duration : 3 Hours Maximum : 75 Marks

Part - A $(10 \times 2 = 20)$

Answer **all** the Questions

1. Define a monoid. Give a non-trivial example of a monoid.

2. If g is a homomorphism from a commutative semi group $(S, *)$ onto a semigroup (Γ, Δ), prove that (Γ, Δ) is also commutative.

3. Let p and q be two given statements. Write the truth table for the statement "p if and only if q".

4. Verify whether $(P \vee Q) \rightarrow P$ is a tautology.

5. Is the statement $(\forall x)(P(x)) \rightarrow (\exists x)(P(x))$ a logically valid statement ? Justify.

6. Verify the validity of the following statements :-
   i) All men are mortal
   ii) Socrates is a man
   iii) Therefore Socrates is a mortal

7. Give an example of a non-modular lattice.

8. If D(n) denotes the lattice of all positive divisors of the integer n, draw the Hasse diagram of (D (32), /).

9. Let a, b, x be elements in a Boolean algebra B. Prove that $a \vee x = b \vee x$, $a \wedge x = b \wedge x \Rightarrow a = b$.

10. Define a Boolean polynomials and give two examples.

Part - B (5 × 5 = 25)

Answer **all** the Questions

11 a. Let N be the set of positive integers and * be the operation defined by a*b = L.C.M. of {a, b}. Is (N, *) a commutative semigroup ? Justify. Is it a monoid ? Justify.

(Or)

b. For any commutative monoid (M, *), Prove that the set of idempotent elements of M forms a submonoid.

12 a. Draw the parsing tree for the formula. $((p \rightarrow (\neg q)) \rightarrow (p \wedge q)$.

(Or)

b. Vefity whether $Q \vee (p \wedge \neg Q) \vee (\neg P \wedge \neg Q)$ is a tautology or a contradiction.

13 a. Is the statement $(\forall x)(P(x)) \vee (\forall x)(Q(x)) \rightarrow (\forall x)(P(x) \vee Q(x))$ a logically valid statement or not ? Justify.

(Or)

b. Show that from (i) $(\exists x)(F(x) \wedge S(x)) \to (\forall y)(M(y) \to W(y))$ and (ii) $(\exists y)(M(y) \wedge \neg W(y))$ the conclusion $(\forall x)(F(x) \to \neg S(x))$ follows.

14 a. Is the lattice of subgroups of A_4 a module lattice ? Justify your answer.

(Or)

b. Show that a lattice L is distributive if, and only if, for all $x, y, z \in L$, $(x \wedge y) \vee (y \wedge z) \vee (z \wedge x) = x \vee y) \wedge (y \vee z) \wedge (z \vee x)$.

15 a. Express the polynomial $p(x_1, x_2, x_3) = x_1 \vee x_2$ in an equivalent sum - of - products canonical form in three variables x_1, x_2, x_3.

(Or)

b. Consider the Boolean function

$$f(x_1, x_2, x_3) = ((x_1 + x_2) + (x_1 + x_3)).\ x_1 . \overline{x_2}.$$

Simplyfy this function and draw the circuit gate diagram for it.

Part - C (3 × 10 = 30)

Answer any **three** Questions

16. a) Let N be the set of positive integers and $S = N \chi N$. Let * be a n operation on S define by $(a, b) * (c, d) = (a + c, b + d)$. Show that S is a semigroup. Define $f : (S, *) \to (Z, +)$ by $f(a, b) = a - b$. Show that f is a homomorphism.

b) Show that the set operation union distributes over intersection.

17. Without using the truth table, find

i) The principal disjunctive normal form of
$(P \wedge Q) \vee (\neg P \wedge R) \vee (Q \wedge R)$.

ii) The principal conjunctive normal form of :-
$P \vee (\neg P \rightarrow (Q \vee (\neg Q \rightarrow R)))$.

18. a) Is the statement $(\forall x)\,(P(x) \wedge Q(x)) \leftrightarrow (\forall x)\,(P(x)) \wedge (\forall x)\,(Q(x))$ a logically valid statement or not ? Justify.

b) Obtain the following implication :-
$(\forall x)\,(P(x) \rightarrow Q(x)),\ (\forall x)\,(R(x) \rightarrow \neg Q(x)) \Rightarrow (\forall x)\,(R(x) \rightarrow \neg P(x))$.

19. a) Show that the set of all positive integers N, ordered by divisibility is a distributive lattice.

b) If G is a group, prove that the set of all normal subgroups of G, forms a modular lattice.

20. a) Let $(B, v, \wedge\, 0, 1, ')$ be a Boolean algebra. Define + and – on B by
$x + y = (x \wedge y^1) \vee (x^1 \wedge y)$ and $x \cdot y = x \wedge y$.
Prove that (B, +, .) is a Boolean ring with unity.

b) Describe the addition of two one-digit binary numbers using switching circuits.

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AF-1907

MMA4E1

M.Sc. DEGREE EXAMINATION, APRIL 2010

Fourth Semester

Mathematics

Elective - AUTOMATA THEORY

(CBCS—2008 Onwards)

Duration : 3 Hours | Maximum : 75 Marks

Part - A $(10 \times 2 = 20)$

Answer **All** the questions

1. Define a finite automation.

2. What is a Transistion System ? When does it accept a string w ?

3. Find the highest type number which can be applied to the production $S \rightarrow Aa$, $A \rightarrow C/Ba$, $B \rightarrow abc$.

4. Define a Grammar and give an example.

5. Let L_1, L_2 be subsets of $\{a, b\}^*$, such that $L_1 \subseteq L_2$. If L_2 is not regular, is it true that L_1 is not regular ? Justify your answer.

6. Construct a context free grammar generating

$$L = \{a^m b^n \mid m > n, m, n \geq 1\}.$$

7. Give a regular expression for representing the set L of strings in which every 0 is immediately followed by atleast two 1's.

8. If X and Y are regular sets over Σ, prove that $X \cap Y$ is also regular over Σ.

9. Let G be the grammar $S \rightarrow OB|1A$, $A \rightarrow O|OS|1AA$, $B \rightarrow 1|1S|OBB$. For the string 0011 0101, find the derivation tree.

10. Let G be a grammar $S \rightarrow SbS|a$. Is G ambiguous ? Justify.

Part - B $(5 \times 5 = 25)$

Answer **All** the questions

11 a. Construct a DFA accepting all strings over {a, b} ending in ab.

(*Or*)

b. Construct a deterministic finite automation equivalent to $M = (\{q_0, q_1, q_2, q_3\}, \{a, b\}, \delta, q_0 \{q_3\})$ where δ is given by

	State / Σ	a	b
$\rightarrow$	q_0	q_0, q_1	q_0
	q_1	q_2	q_1
	q_2	q_3	q_3
	O_3		q_2

12 a. Find a grammar generating $L = \{a^n, b^n, c^j \mid n \geq 1, j \geq 0\}$.

(*Or*)

b. Prove that every monotonic grammar G is equivalent to a type 1 grammar.

13 a. Construct context - free grammars to generate

i) $\{0^m 1^n \mid m \neq n, m, n \geq 1\}$.

ii) The set of all strings over $\{0, 1\}$ containing twice as many 0's is 1's.

(*Or*)

b. Prove that each of the classes $\mathcal{L}_0, \mathcal{L}_{cfl}, \mathcal{L}_{csl}, \mathcal{L}_{rl}$ is closed under concatenation.

14 a. State and prove pumping Lemma for regular sets.

(*Or*)

b. Construct a regular grammar accepting $L = \{w \in \{a, b\}^* \mid$ w is a string over $\{a, b\}$ such that the number of b's is 3 mod 4$\}$.

15 a. Find a reduced grammar equivalent to the grammar $S \rightarrow aAa$, $A \rightarrow Sb \mid bcc \mid DaA$, $C \rightarrow abb \mid DD$, $E \rightarrow ac$, $D \rightarrow aDa$.

(*Or*)

b. For every context-free grammar G_1, Prove that there is an equivalent grammar G_2 in Chomsky normal form.

Part - C $(3 \times 10 = 30)$

Answer any **Three** questions

16. If L is a set accepted by an NDFA, prove that there exists a DFA which also accepts L.

17. Construct a grammar G generating $\{xx \mid x \in \{a, b\}^*\}$.

18. Prove that a context - sensitive language is recursive.

19. Prove that any set L accepted by a finite automation M can be represented by a regular expression.

20. Reduce the grammar G given by $S \rightarrow aAD$, $A \rightarrow aB \mid bAB$, $B \rightarrow b$, $D \rightarrow d$ in Chomsky Normal form.

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AF-1908	MMA4E2

M.Sc. DEGREE EXAMINATION, APRIL 2010

Fourth Semester

Mathematics

Elective - DATA STRUCTURES AND ALGORITHMS

(CBCS—2008 Onwards)

Duration : 3 Hours Maximum : 60 Marks

Part - A $(10 \times 1^1/_2 = 15)$

Answer **All** the questions

1. Write a function SWAP to interchange the values of two integer variables.

2. State any two uses of the operator 'new'.

3. Let L = {a, b, c, d} be a linear list. What are the result of the operations

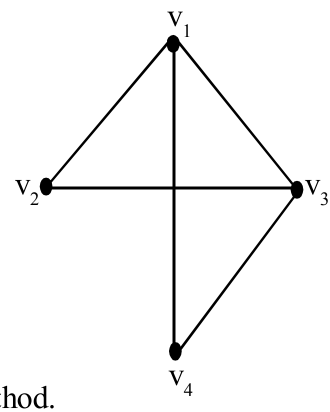
   (i) insert (2, f),

   (ii) get (2).

4. Suppose that a 500 × 500 matrix that has 2000 non-zero terms is to be represented. How much space is needed when sparse matrix is used ?

5. Draw a binary expression tree to represent the expression $(a * b) + (c / d)$.

6. Define spanning tree of a graph. Find the spanning tree of the graph.



7. Define Greedy method.

8. Define Divide and Conquer method.

9. Write a recursive function for Knapsack problem.

10. Define branch and bound technique.

Part - B $(5 \times 3 = 15)$

Answer **All** the questions

11 a. Explain the components of space complexity.

(*Or*)

b. Explain the components of Time complexity.

12 a. Write a function change Length 2D to change the length of a two dimensional array. You must allow for a change in both dimensions of the array. Test your code.

(*Or*)

b. Write a method to multiply two sparse matrices represented using an array linear list.

13 a. Write a function to determine the height of a linked binary tree. What is the time complexity of your function ?

(*Or*)

b. Explain with an example the method of deleting an element from an AVL search tree.

14 a. Show that the greedy algorithm for the change-making problem generates change with the fewest number of coins.

(*Or*)

b. Prove that the average complexity of quicksort is $\theta(n \log n)$ where n is the number of elements to be sorted.

15 a. Explain the method of solving a shortest path problem using dynamic programming method.

(*Or*)

b. Use the dynamic programming recurrence for the matrix chains problem to compute ((i, j) and kay (i, j) $1 \leq i \leq j \leq q$ for q = 5 and r = (100, 10, 100, 2, 50, 4). What is the cost of the best way to multiply the q matrices ?

Part - C $(3 \times 10 = 30)$

Answer any **Three** questions

16. Write a method minmax to determine the locations of the minimum and maximum elements in an array A. Let n be the instance characteristic. What is the number of comparisons between the elements of A ? What are the best-case and worst case numbers of comparisons between elements of A ? What can you say about the expected relative performance of the two functions ?

17. Develop a C++ class two stacks in which a single array is used to represent two stacks. Peg the bottom of one stack at position 0 and the bottom of the other at position array length – 1. The two stacks grow toward the middles of the array. The class must contain methods to perform all the operations of the ADT stack on each of the two stacks.

18. Develop an inorder iterator for the class linked Binary Tree. The time taken to enumerate all the elements of an n-element binary tree should be 0(n). The complexity of no method should excess 0(h), where h is the tree height, and the space requirements should be 0(h). Test your code.

19. Write a complete program for the defective chess board problem.

20. Write a program to implement the Bellman-Ford algorithm that finds the shortest path.

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AF-1882	MMA1C1

M.Sc. DEGREE EXAMINATION, APRIL 2010

First Semester

Mathematics

ALGEBRA - I

(CBCS—2008 Onwards)

Duration : 3 Hours Maximum : 75 Marks

Part - A ($10 \times 2 = 20$)

Answer **All** questions

1. G is a group. H and K are subgroups of G such that $0(H) = 4$ and $0(k) = 9$. Find $0(HK)$, if HK is a subgroup of G.

2. If ϕ is a homomorphism of a group G into a group G' with Kernel K, prove that K is a normal subgroup of G.

3. Prove that $a \in Z$ if, and only if, $N(a) = G$.

4. Can a group of order 28 be simple ? Justify.

5. Prove that every field is a integral domain.

6. Let R be a ring, $a \in R$ and let $r(a) = \{x \in R \mid ax = 0\}$. Prove that $r(a)$ is a right ideal of R.

7. Prove that any homomorphism of a field is either an isomorphism or takes each element into zero.

8. If U is an ideal of R and $1 \in U$ prove that $U = R$.

9. Prove that $x^2 + x + 1$ is irreducible over the field of integers mod 2.

10. If f(x), g(x) are two non-zero elements of F[x], prove that deg (f(x), g(x)) = deg f(x) + deg g(x).

Part - B $(5 \times 5 = 25)$

Answer **All** questions

11 a. Let ϕ be a homomorphism of G onto $\bar{G}$ with kernel K. Let $\bar{N}$ be a normal subgroup of G, $N = \{x \in G \mid \phi(x) \in \bar{N}\}$. Prove that G/N is isomorphic onto (G/K) / (N/K).

(*Or*)

b. If H and K are finite subgroups of G of orders 0(H) and 0(K) respectively, prove that $0(HK) = \dfrac{0(H) \, . \, 0(K)}{0(H \cap K)}$

12 a. Prove that the number of p-Sylow subgroups in a group G for a given prime p is of the form 1 + Kp and it divides 0(G).

(*Or*)

b. Let G be a group and suppose that G is the internal direct product of $N_1, N_2, \ldots, N_k$. Let $T = N_1 \times N_2 \times \ldots \times N_k$. Prove that G and T are isomorphic.

13 a. Prove that the set of integers mode7 under the addition mod7 and multiplication mod7 is a field.

(*Or*)

b. Prove that every finite integral domain is a field.

14 a. If R is a commutative ring with unit element and M is an ideal of R prove that M is a maximal ideal of R if and only if, R/M is a field.

(*Or*)

b. Let R be a commutative ring with unit element. Prove that R is a field if and only if, its only ideals are $\{ 0 \}$ and R.

15 a. State and prove the Eisenstein's criterion on irreducibility of polynomials over the rationals.

(*Or*)

b. Prove that the ring of Gaussian integers J[i] is a Euclidean ring.

Part - C $(3 \times 10 = 30)$

Answer any **Three** questions

16. Let G be a group, H a subgroup of G and S be the set of all right cosets of H and G. Prove that there is a homomorphism θ of G into A(S) and the Kernel of θ is the largest normal subgroup of G which is contained in H.

17. If p is a prime number and p/o(G), can G have a subgroup of order p ? Justify your answer.

18. Let $Q = \{ \alpha_0 + \alpha_1 i + \alpha_2 j + \alpha_3 k \mid \alpha_0, \alpha_1, \alpha_2, \alpha_3$ are all real numbers.$\}$ Prove that Q is a skew field with a suitably defined addition and multiplication.

19. Can we imbed an integral domain into a field ? Justify your answer.

20. Let R be a Euclidean ring and $A = (a_0)$ be an ideal of R. Prove that A is a maximal ideal of R if, and only if, a_0 is a prime element of R.

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