PHY

1801

GS-2010 (Physics)

Full Name : ____________________

Reference Code : ____________________

TATA INSTITUTE OF FUNDAMENTAL RESEARCH

Written Test in **PHYSICS**
December 13, 2009

Duration : Three hours (3 hours)

Please read all instructions carefully before you attempt the questions.

1. Write your FULL NAME and REFERENCE CODE in block letters on this page and also fill-in all details on the ANSWER SHEET.

2. The Answer Sheet is machine-readable. Please read the instructions on the reverse of the answer sheet before you start filling it up. Only use HB pencils to fill-in the answer sheet.

3. This test comes in two sections (A & B), both of which contain multiple choice-type questions. Indicate your ANSWER ON THE ANSWER SHEET by blackening the appropriate circle for each question. Only one of the four/six answers given at the end of each question is correct.
Do not mark more than one circle for any question : this will be treated as a wrong answer.

4. Section A contains 20 questions. Each correct answer gets you +3 marks and a wrong answer -1 mark. Section B has 8 questions. Each correct answer in Section B gets you +5 and a wrong answer - 1 mark. The maximum marks is 100.

5. A rough guideline for time to be spent: approximately 5 minutes for each question in Section A, and about 10 minutes for each question in Section B.

6. We advise you to first mark the correct answers in the QUESTION SHEET and then to TRANSFER these to the ANSWER SHEET only when you are sure of your choice.

7. Rough work may be done on blank pages of the question paper. If needed, you may ask for extra rough sheets from an Invigilator.

8. **Use of calculators is permitted. Calculator which plots graphs is NOT allowed.** A list of useful physical constants and conversion factors are given on the next page.

9. Do NOT ask for clarifications from the invigilators regarding the questions. They have been instructed not to respond to any such inquiries from candidates. In case a correction/clarification is deemed necessary, the invigilator(s) will announce it publicly.

10. **This set of question paper must be returned along with your answer sheet and all extra rough sheets used.**

Useful Constants

Symbol	Definition/Name	Value
Å	Ångstrom	10^{-10} m
bar	standard atmospheric pressure	1.01×10^{5} Pa
c	speed of light in vacuum	3.0×10^{8} m s^{-1}
e	electron charge (magnitude)	1.60×10^{-19} C
g	acceleration due to gravity (sea level)	9.8 m s^{-2}
G	gravitational constant	6.67×10^{-11} m^{3} kg^{-1}s^{-2}
ϵ_0	permittivity of free space	8.854×10^{-12} F m^{-1}
μ_0	permeability of free space	$4\pi \times 10^{-7}$
a.m.u.	unified atomic mass unit	931.494 MeV/c^{2}
$\hbar$	Planck's constant ($h/2\pi$)	1.05×10^{-34} J s
k	Boltzmann constant	8.62×10^{-5} eV K^{-1} = 1.38×10^{-23} J K^{-1}
m_e	electron rest mass	0.5 MeV/c^{2}
m_n	neutron rest mass	939.6 MeV $\approx 2000\, m_e \approx m_p$
m_p	proton rest mass	938.3 MeV $\approx 2000\, m_e \approx m_n$
N_A	Avogadro number	6.02×10^{23}
$R = kN_A$	gas constant	8.314 J mol^{-1} K^{-1}
$\gamma = C_P/C_V$	ratio of specific heats : monatomic gas	1.67
	: diatomic gas	1.40
R_e	radius of earth	6400 km
R_s	radius of sun	700 000 km
σ	Stefan-Boltzmann constant	5.67×10^{-8} W·m^{-2} K^{-4}
$\hbar c$	conversion constant	197 MeV fm = 3.16149×10^{-17} erg cm
$\alpha = e^2/4\pi\epsilon_0\hbar c$	fine structure constant	1/137
$a_0 = 4\pi\epsilon_0\hbar^2/e^2 m_e$	Bohr radius	0.51 Å
1 eV	electron volt	1.60×10^{-19} J
1 T	Tesla	10^{4} Gauss (G)
1 W	Watt	1 J s^{-1} = 10^{7} erg s^{-1}
1 Pa	Pascal	1 Pa = 10 dyne cm^{-2}
	ionisation energy of hydrogen atom	13.6 eV

Test 2009

A SECTION : 20 × 3 = 60 Marks

Recommended Time : approx. 100 minutes

A1. The matrix

$$\begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix}$$

can be related by a similarity transformation to the matrix

(a) $\begin{pmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & 0 \end{pmatrix}$ (b) $\begin{pmatrix} 2 & 1 & 0 \\ 1 & -1 & -1 \\ 0 & -1 & 2 \end{pmatrix}$

(c) $\begin{pmatrix} 1 & -1 & 0 \\ -1 & 1 & 1 \\ 0 & 1 & -1 \end{pmatrix}$ (d) $\begin{pmatrix} 1 & -1 & 0 \\ -1 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix}$

A2. A car tyre is slowly pumped up to a pressure of 2 atmospheres in an environment at 15° C. At this point, it bursts. Assuming the sudden expansion of the air (a mixture of O_2 and N_2) that was inside the tyre to be adiabatic, its temperature after the burst is

(a) −55° C (b) −37° C (c) −26° C (d) +9° C

A3. A small meteor approaches the Earth. When it is at a large distance, it has velocity v_∞ and impact parameter b. If R_e is the radius of the Earth and v_0 is the escape velocity, the condition for the meteor to strike the Earth is

(a) $b < R_e\sqrt{1-(v_0/v_\infty)^2}$ (b) $b > R_e\sqrt{1+(v_0/v_\infty)^2}$

(c) $b < R_e\sqrt{1+(v_0/v_\infty)^2}$ (d) $b = R_e\,(v_0/v_\infty)$

A4. Consider a very, very thin wire of uniformly circular cross section. The diameter of the wire is of the order of microns. The correct equipment required to measure the precise value of *resistivity* of this wire is

(a) ammeter, voltmeter, scale, slide calipers

(b) ammeter, magnet, screw gauge, thermometer

(c) voltmeter, magnet, screw gauge, scale

(d) ammeter, voltmeter, scale, monochromatic laser source

A5. A function $f(x)$ is defined in the range $-1 \leq x \leq 1$ by

$$f(x) = \begin{matrix} 1-x & \text{for} & x \geq 0 \\ 1+x & \text{for} & x < 0 \end{matrix}$$

The first few terms in the Fourier series approximating this function are

(a) $\frac{1}{2} + \frac{4}{\pi^2} \cos \pi x + \frac{4}{9\pi^2} \cos 3\pi x + \ldots$

(b) $\frac{1}{2} + \frac{4}{\pi^2} \sin \pi x + \frac{4}{9\pi^2} \sin 3\pi x + \ldots$

(c) $\frac{4}{\pi^2} \cos \pi x + \frac{4}{9\pi^2} \cos 3\pi x + \ldots$

(d) $\frac{1}{2} - \frac{4}{\pi^2} \cos \pi x + \frac{4}{9\pi^2} \cos 3\pi x - \ldots$

A6. A lead container contains 1 gm of a $^{60}_{27}$Co radioactive source. It is known that a $^{60}_{27}$Co nucleus emits a β particle of energy 316 KeV followed by two γ emissions of energy 1173 and 1333 KeV respectively. Which of the following experimental methods would be the best way to determine the lifetime of this $^{60}_{27}$Co source ?

(a) Measure the change in temperature of the source

(b) Measure the weight of the source now and again after one year

(c) Measure the recoil momentum of the nucleus during β emission

(d) Measure the number of γ photons emitted by this source

A7. A beam of hydrogen molecules travels in the z direction with a kinetic energy of 1 eV. The molecules are in an excited state, from which they decay and dissociate into two hydrogen atoms. When one of the dissociated atoms has its final velocity perpendicular to the z direction, its kinetic energy is always 0.8 eV. The energy released in the dissociative reaction is

(a) 0.26 eV (b) 2.6 eV (c) 0.36 eV (d) 3.6 eV

A8. Two parallel plates of metal sandwich a dielectric pad of thickness d, forming an ideal capacitor of capacitance C. The dielectric pad is elastic, having a spring constant k. If an ideal battery of voltage V across its terminals is connected to the two plates of this capacitor, the fractional change $\delta d/d \ll 1$ in the gap between the plates is

(a) zero (b) $+\frac{\frac{1}{2}CV^2}{kd^2}$

(c) $-\frac{\frac{1}{2}CV^2}{kd^2+CV^2}$ (d) $-\frac{\frac{1}{2}CV^2}{kd^2-CV^2}$

A9. When white light is scattered from a liquid, a strong absorption line is seen at 400 nm, and two emission lines are observed, one of which is at 500 nm, and another in the infra-red portion of the spectrum. The wavelength of this second emission line is

(a) 900 nm (b) 2000 nm (c) 100 nm (d) 222 nm

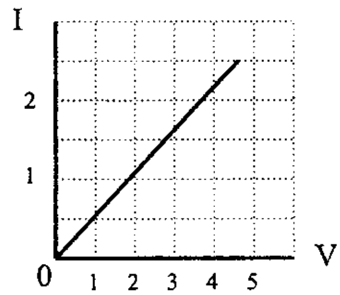
A10. A detector is used to count the number of γ rays emitted by a radioactive source. If the number of counts recorded in exactly 20 seconds is 10000, the error in the counting rate per second is

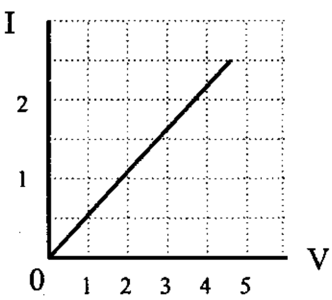
(a) $\pm$ 5.0 (b) $\pm$ 22.4 (c) $\pm$ 44.7 (d) $\pm$ 220.0

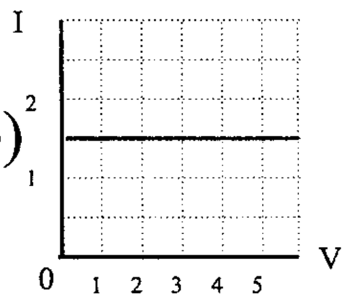
A11. Consider a standard chess board with 8×8 squares. A piece starts from the lower left corner, which we shall call Square (1,1). A single move of this piece corresponds to <u>either</u> one step right, i.e. to Square (1,2) <u>or</u> one step forwards, i.e. to Square (2,1). If it continues to move according to these rules, the number of different paths by which the piece can reach the Square (5,5) starting from the Square (1,1) is

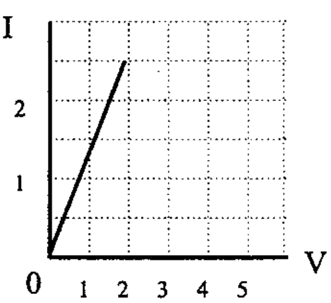
(a) 120 (b) 72 (c) 70 (d) 45

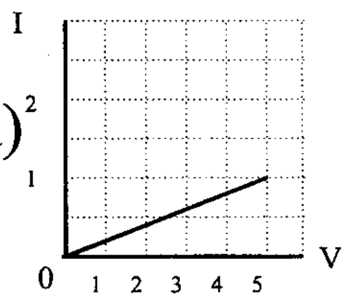
A12. The uppermost graph in the set below shows the variation of current v/s voltage applied across a copper conductor at temperature T_1. Which of the graphs below — marked (a), (b), (c) or (d) — will show the possible variation of the I–V curve for the same conductor at another temperature $T_2 > T_1$?



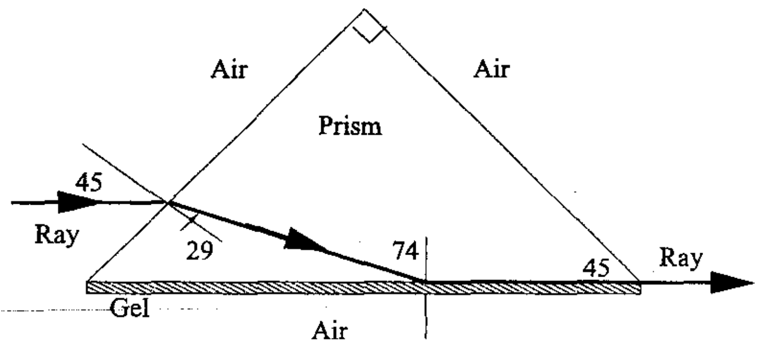
(a)


(b)


(c)


(d)


A13. A ray of light is incident on a right-angled prism as shown in the figure below. The lower surface of this prism is coated with a gel. If the incident ray makes angles (marked in degrees) as shown in the figure, the refractive index of the gel must be



(a) 1.40 (b) 1.46 (c) 1.50 (d) 1.52

A14. A particle P_1 is confined in a one-dimensional infinite potential well with walls at $x = \pm 1$. Another particle P_2 is confined in a one-dimensional infinite potential well with walls at $x = 0, 1$. Comparing the two particles, one can conclude that

(a) the no. of nodes in the n^{th} excited state of P_1 is twice that of P_2

(b) the no. of nodes in the n^{th} excited state of P_1 is half that of P_2

(c) the energy of the n^{th} level of P_1 is the same as that of P_2

(d) the energy of the n^{th} level of P_1 is one quarter of that of P_2

A15. A charged particle is in the ground state of a one-dimensional harmonic oscillator potential, generated by electrical means. If the power is suddenly switched off, so that the potential disappears, then, according to quantum mechanics,

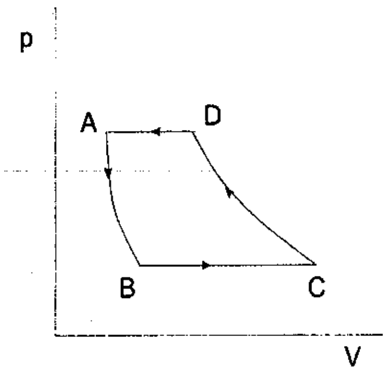
(a) the particle will shoot out of the well and move out towards infinity in one of the two possible directionas

(b) the particle will stop oscillating and as time increases it may be found farther and farther away from the centre of the well

(c) the particle will keep oscillating about the same mean position but with increasing amplitude as time increases

(d) the particle will undergo a transition to one of the higher excited states of the harmonic oscillator

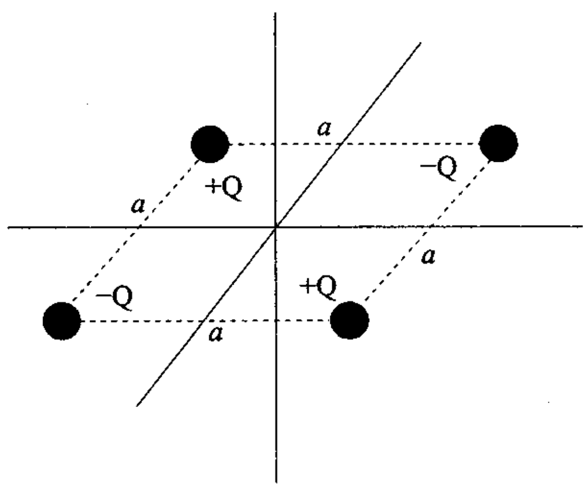
A16. The pV diagram given below represents a



(a) Carnot refrigerator (b) Carnot engine

(c) gas turbine refrigerator (d) gas turbine engine

A17. In the laboratory, four point charges $+Q, -Q, +Q, -Q$ are placed at the four ends of a horizontal square of side a, as shown in the figure below. The number of neutral points (where the electric field vanishes) is

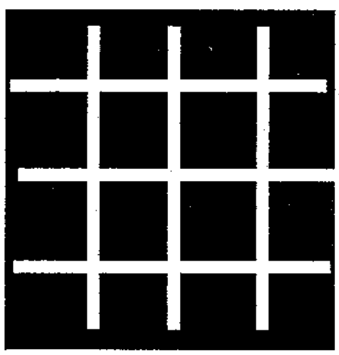


(a) ∞ (b) 4 (c) 1 (d) zero

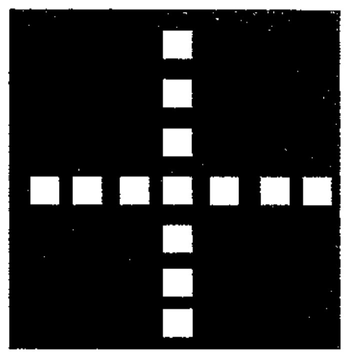
A18. Coherent monochromatic light falling through a small aperture produced a Fraunhofer diffraction pattern as shown below

By looking at this diffraction pattern carefully one can guess that the shape of the aperture was

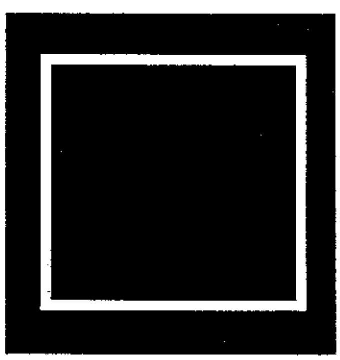
(a)



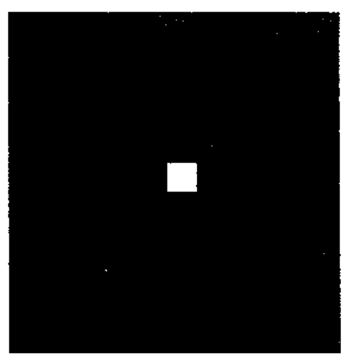
(b)



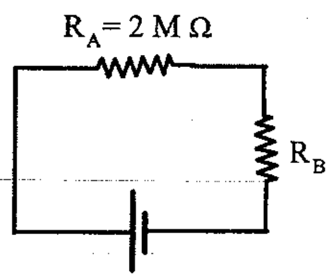
(c)



(d)

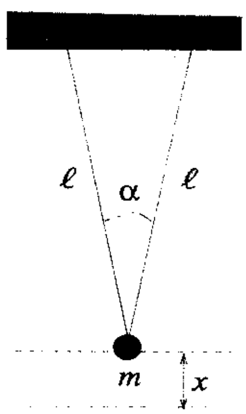


A19. in the circuit given below, a person measures 9.0 V across the battery, 3.0 V across the 2 MΩ resistor R_A and 4.5 V across the unknown resistor R_B, using an ordinary voltmeter which has a finite input resistance r. Assuming that the battery has negligible internal resistance, it follows that (*i*) the resistance R_B and (*ii*) the input resistance r of the voltmeter are, in MΩ,



(a) $R_B = 3.0$, $r = 6.0$
(b) $R_B = 2.5$, $r = 7.5$
(c) $R_B = 4.0$, $r = 12.0$
(d) $R_B = 4.5$, $r = 10.0$

A20. A heavy mass m is suspended from two identical steel wires of length ℓ, radius r and Young's modulus Y, as shown in the figure below. When the mass is pulled down by a distance x ($x \ll \ell$) and released, it undergoes elastic oscillations in the vertical direction with a time period



(a) $\frac{2\pi}{r}\sqrt{\frac{m\ell}{2Y\cos^2(\alpha/2)}}$
(b) $2\pi\sqrt{\frac{\ell\cos(\alpha/2)}{g}}$
(c) $\sqrt{\frac{2\pi m\ell}{Yr^2}}$
(d) $\frac{2\pi}{r}\sqrt{\frac{mg\ell}{2Y}}$

B SECTION: 8 × 5 = 40 Marks

Recommended Time : approx. 80 minutes

B1. The wave function Ψ of a quantum mechanical system described by a Hamiltonian $\hat{H}$ can be written as a linear combination of Φ_1 and Φ_2 which are the eigenfunctions of $\hat{H}$ with eigenvalues E_1 and E_2 respectively. At $t = 0$, the system is prepared in the state $\Psi_0 = \frac{4}{5}\Phi_1 + \frac{3}{5}\Phi_2$ and then allowed to evolve with time. The wavefunction at time $T = \frac{1}{2}h/(E_1 - E_2)$ will be (accurate to within a phase)

(a) $\frac{4}{5}\Phi_1 + \frac{3}{5}\Phi_2$ (b) Φ_1 (c) $\frac{4}{5}\Phi_1 - \frac{3}{5}\Phi_2$
(d) Φ_2 (e) $\frac{3}{5}\Phi_1 + \frac{4}{5}\Phi_2$ (f) $\frac{3}{5}\Phi_1 - \frac{4}{5}\Phi_2$

B2. Light transmitted along an optical fibre incurs losses due to Rayleigh scattering from inhomogeneities. If a fibre of given length transmits 50% of the monochromatic light coupled into it at a wavelength of 1350 nm, the transmitted fraction for the same fibre at 1550 nm will be

(a) 55% (b) 57% (c) 62% (d) 67% (e) 74% (f) 87%

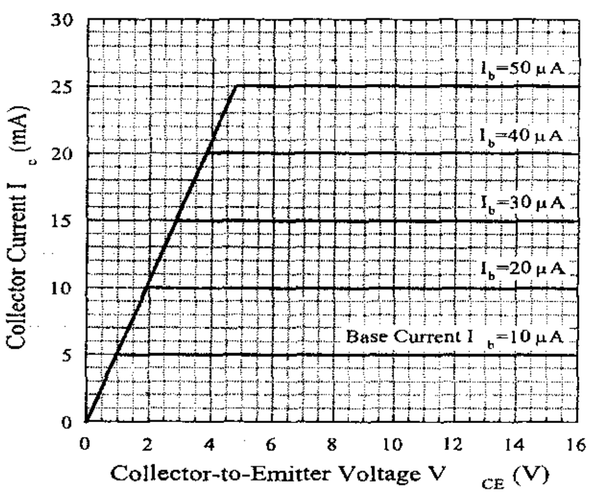
B3. A quantum system has three energy levels -0.12 eV, -0.2 eV and -0.44 eV respectively. Three electrons are distributed among these levels. At a temperature of 1727^0 C the system has total energy -0.68 eV. The free energy of the system is approximately

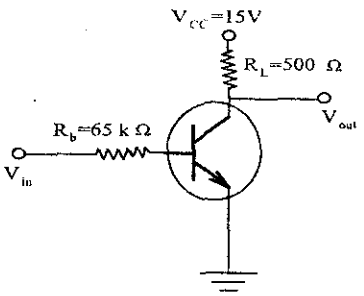
(a) $+1.5$ eV (b) $+0.3$ eV (c) -0.1 eV
(d) -0.3 eV (e) -1.0 eV (f) -1.5 eV

B4. An atom is capable of existing in two states: a ground state of mass M and an excited state of mass $M + \Delta$. If the transition from the ground state to the excited state proceeds by the absorption of a photon, the photon frequency in the laboratory frame (where the atom is initially at rest) is

(a) $\frac{\Delta c^2}{h}$ (b) $\frac{\Delta c^2}{h}\left(1 + \frac{\Delta}{2M}\right)$ (c) $\frac{Mc^2}{h}$
(d) $\frac{\Delta c^2}{h}\left(1 - \frac{\Delta}{2M}\right)$ (e) $\frac{Mc^2}{h}\left(1 + \frac{\Delta}{2M}\right)$ (f) $\frac{Mc^2}{h}\left(1 - \frac{\Delta}{2M}\right)$

B5. A plot of the common-emitter characteristics of a silicon n-p-n transistor is shown below. Given this information, and assuming that there will be a 0.7 V drop across a forward biased silicon p-n junction, the approximate value of the output voltage V_{out} for an input voltage $V_{in} = 2$ V in the adjacent circuit will be





(a) 4 V (b) 6 V (c) 8 V (d) 10 V (e) 12 V (f) 14 V

B6. Measurement of the electric field (E) and the magnetic field (B) in a plane-polarized electromagnetic wave in vacuum led to the following:

$$\frac{\partial E}{\partial x} = \frac{\partial E}{\partial y} = 0 \qquad \frac{\partial E}{\partial z} = -\frac{\partial B}{\partial t}$$

$$\frac{\partial B}{\partial x} = \frac{\partial B}{\partial y} = 0 \qquad \frac{\partial B}{\partial z} = +\frac{\partial E}{\partial t}$$

It follows that

(a) $\vec{E} = E\hat{\imath}$, $\vec{B} = B\hat{\jmath}$ and the wave was travelling along $\hat{k}$

(b) $\vec{E} = E\hat{\jmath}$, $\vec{B} = B\hat{\imath}$ and the wave was travelling along $\hat{k}$

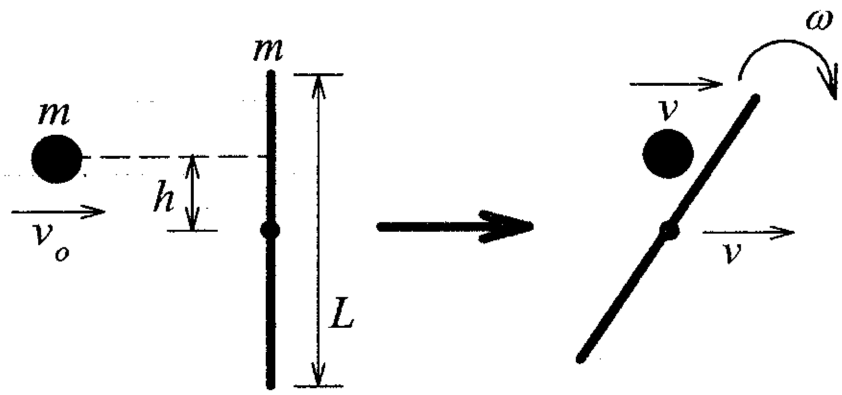
(c) $\vec{E} = E\hat{\jmath}$, $\vec{B} = B\hat{k}$ and the wave was travelling along $-\hat{\imath}$

(d) $\vec{E} = E\hat{k}$, $\vec{B} = B\hat{\imath}$ and the wave was travelling along $\hat{\jmath}$

(e) $\vec{E} = E\hat{\imath}$, $\vec{B} = B\hat{k}$ and the wave was travelling along $-\hat{\jmath}$

(f) the wave was travelling along $\pm\hat{k}$ but directions of $\vec{E}$ and $\vec{B}$ are not uniquely defined

B7. A mass m travels in a straight line with velocity v_0 perpendicular to a uniform stick of mass m and length L, which is initially at rest. The distance from the centre of the stick to the path of the travelling mass is h (see figure). Now the travelling mass m collides elastically with the stick, and the centre of the stick and the mass m are observed to move with equal speed v after the collision. Assuming that the travelling mass m can be treated as a point mass, and the moment of inertia of the stick about its center is $I = \frac{mL^2}{12}$, it follows that the distance h must be



(a) $\frac{L}{2}$ (b) $\frac{L}{4}$ (c) $\frac{L}{\sqrt{6}}$ (d) $\frac{L}{\sqrt{3}}$ (e) $\frac{L}{3}$ (f) zero

B8. The binding energy per nucleon for ^{235}U is 7.6 MeV. The ^{235}U nucleus undergoes fission to produce two fragments, both having binding energy per nucleon 8.5 MeV. The energy released, in Joules, from the complete fission of 1 Kg of ^{235}U is, therefore,

(a) 8000 (b) 10^{35} (c) 450

(d) 20000 (e) 8.7×10^{13} (f) 5.0×10^{8}